Exercise 1: Analytical Integration

Consider the functions
\[ f(x) = -4x^2 - 2 \]
and
\[ g(x) = \left( -16x^3 + 40x^2 - 35x + 11 \right) \]

Compute the antiderivatives and evaluate the integrals.

Exercise 2: Composite Trapezoidal Rule

Compute the antiderivatives and evaluate the integrals.

\[ g(x) = \int f(x) \, dx \]
\[ g(x) = \int (x^3 - 12x^2 + 45x - 27) \, dx \]
\[ g(x) = \int (x^3 - 12x^2 + 45x - 27) \, dx \]
\[ g(x) = \int (x^3 - 12x^2 + 45x - 27) \, dx \]

Exercise 3: Composite Simpson Rule

On the same axis as Exercise 2 for the Composite Simpson Rule.

Exercise 4: Archimedes' Hierarchical Approach

In this exercise we will use Archimedes’ approach to approximate the integral. Let \( E \subseteq [a, b] \) be a subset of \( \mathbb{R} \) and \( \varepsilon > 0 \) be a vector of function values with \( \varepsilon_i = \frac{\varepsilon}{2^{i-1}} \).

Write a function that transforms a given vector \( u \) to a similar vector \( v \) containing the hierarchical coefficients according to the definition above and \( n \) being the number of trapezoids used.

Write a function that approximates the integral via the Composite Trapezoidal Rule. Complete the function template.

Write a function that transforms \( (2^l-1) \) functional values in \( u \) to hierarchical coefficients.

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