

# Fundamental Algorithms

## Exercise 1

Write an algorithm that copies all keys that are stored in a binary tree into an array of appropriate size. In the resulting array, the keys shall be sorted in descending order.

## Exercise 2

Prove or disprove the following statement:

If, in a binary search tree, the keys  $k_1$  and  $k_2$  are deleted, the resulting search tree does not depend on whether  $k_1$  or  $k_2$  is deleted first (i.e. deleting  $k_1$  before  $k_2$  will lead to the same search tree as deleting  $k_2$  before  $k_1$ ).

## Exercise 3

Show that an AVL tree containing  $m$  inner nodes\* that have exactly two non-empty sons, has at most  $\frac{\sqrt{5}-1}{2}m$  nodes with a height balance different from 0.

\* *inner nodes* are nodes that are not leaves.

## Exercise IV (CSE)

Consider the binary tree given by the expression

```
x = (5, (3, emptyTree, (4, emptyTree, emptyTree)),
      (8, (6, emptyTree, emptyTree), (10, (9, emptyTree, emptyTree),
      (13, emptyTree, emptyTree))))
```

- draw a diagram of this binary tree and decide whether its a binary search tree
- perform the following operations (using the resp. algorithms from the lectures), and draw a diagram of the search tree after each operation:
  - TREE\_INSERT(x, 11)
  - TREE\_DELETE(x, 5)
  - TREE\_INSERT(x, 5)
  - TREE\_INSERT(x, 12)

### Exercise V (CSE)

Decide whether the binary tree given in exercise IV is an AVL tree

- before the insert/delete operations, and
- after each of the regular insert/delete operations.

Again, perform the insert/delete operations given in exercise IV, and name and perform the rotation(s) to restore the AVL property after each step (if required). Draw a diagram of the search tree after each of your insert/delete, or rotation operations.