

Fundamental Algorithms

Exercise 1

(a) Compare the growth of the following functions using the o -, O -, and Θ -notation:

1. $n \log n$
2. n^l for all $l \in \mathbb{N}$
3. 2^n

(b) Try to give a simple characterization of the growth of the following expressions using the Θ -notation:

$$1) \sum_{i=1}^n \frac{1}{i} \qquad 2) \log(n!)$$

Hint for $\log(n!)$: try to prove $n^{\frac{n}{2}} \leq n! \leq n^n$ first!

Exercise 2

Let $l(x)$ be the number of bits of the representation of x in the binary system. Prove:

$$\sum_{i=1}^n l(i) = \Theta(n \log n)$$

Exercise 3

Prove that Θ defines an equivalence relation on the set of functions $\{f \mid f: \mathbb{N} \rightarrow \mathbb{R}\}$. Use that $(f, g) \in \Theta \Leftrightarrow f \in \Theta(g)$