

Fundamental Algorithms

Exercise 1

Consider the following RAM program:

```
0  R0 ← 1
1  R1 ← 8
2  R2 ← 1
3  R8 ← R8 + R0
4  R8 ← R8 - R0
5  R3 ← R8 - R2
6  IF R3 = 0 GOTO 24
7  R4 ← R2
8  R5 ← R2 + R0
9  R3 ← R5 - R8
10 IF R3 > 0 GOTO 4
11 R4 ← R4 + R1
12 R5 ← R5 + R1
13 R6 ← RR4
14 R7 ← RR5
15 R3 ← R6 - R7
16 IF R3 = 0 GOTO 19
17 RR4 ← R7
18 RR5 ← R6
19 R4 ← R4 - R1
20 R5 ← R5 - R1
21 R4 ← R4 + R0
22 R5 ← R5 + R0
23 GOTO 9
```

For an input sequence (x_1, x_2, \dots, x_n) , the starting configuration of the RAM shall be given by:

$$\begin{aligned} \langle Ri \rangle &:= 0 && \text{for } 0 \leq i \leq 7 \quad \text{and} \quad i \geq n + 9 \\ \langle R8 \rangle &:= n \\ \langle Ri \rangle &:= x_{i-8} && \text{for } 9 \leq i \leq n + 8 \\ \langle PC \rangle &:= 0 \end{aligned}$$

After the RAM has finished, registers R9 to R($n + 8$) contain the input sequence in sorted (ascending) order.

Exercises:

- (a) Execute the RAM program for the input sequence $(3, 7, 3, 2)$, and denote the contents of the registers for each step (steps where different registers are modified one after another may be combined into one step).
- (b) What is the task of register R5?
- (c) The given RAM program is a variant of one of the sorting algorithms discussed in the lectures. State the name of this algorithm, and discuss how this version differs from the version given in the lectures.

Exercise 2

Show that $V_{\text{Merge}}^{\text{rek}}(n) = n \cdot \lceil \log n \rceil - 2^{\lceil \log n \rceil} + 1$ will solve the MERGESORT recurrence

$$\begin{aligned} V_{\text{Merge}}^{\text{rek}}(1) &= 0, \\ V_{\text{Merge}}^{\text{rek}}(n) &= V_{\text{Merge}}^{\text{rek}}\left(\left\lceil \frac{n}{2} \right\rceil\right) + V_{\text{Merge}}^{\text{rek}}\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1 \quad \text{for } n > 1 \end{aligned}$$