Exercise 1

Try the Recursion Tree Method (compare lecture) for the following recurrence:

\[ T(n) = T(n/3) + T(2n/3) + O(n) \]

Show that the height of the recursion tree is in \( O(\log(n)) \).

What could be a flaw using the recursion tree method for such unbalanced trees?
Show that \( T(n) \in O(n \log(n)) \), anyway, by using the substitution method.

Exercise 2

Consider a partitioning algorithm that, in the worst case, will partition an array of \( m \) elements into two partitions of size \( \lfloor \epsilon m \rfloor \) and \( \lceil (1 - \epsilon) m \rceil \), where \( \epsilon \) is fixed, and \( 0 < \epsilon < 1 \). Show that a quicksort algorithm based on this partitioning has a worst-case complexity of \( O(n \log n) \).

*Hint or solution: solve the recurrence by guessing the solution and finding the involved constants.*