**Introduction to Scientific Computing**

**Exercise 3: Systems of linear differential equation**

For each of the systems given in 1.) through 3.)

a.) Find the eigenvalues.

b.) Classify the critical point \((0, 0)\).

c.) Sketch several trajectories.

\[
1.) \quad \frac{d\vec{x}}{dt} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{x}
\]

\[
2.) \quad \frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \vec{x}
\]

\[
3.) \quad \frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \vec{x}
\]

4.) The differential equation describing the motion of a pendulum (see slides of the lecture of Octobre 29th) is

\[
\frac{d^2\alpha}{dt^2} + k \cdot \frac{d\alpha}{dt} + c \cdot \alpha = 0,
\]

where \(k\) is nonnegative and \(c\) is positive. Write this second order equation as a system of two first order equations for \(x = \alpha\) and \(y = \frac{d\alpha}{dt}\). Show that \(x = 0, y = 0\) is a critical point, and analyse the nature and stability as a function of the parameters \(c\) and \(k\). What follows for the motion of the pendulum?