Technische Universität München WS 2003/04
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Introduction to Scientific Computing

Exercise 12: Finite-Element Method

The Laplace equation

$$u''(x) = 0 \qquad x \in [0,1], \quad u(0) = u(1) = 0$$

shall be solved by a finite-element approach.

A classical Galerkin method based on linear shape functions (see lecture) is to be applied. The n elements are of equal length $1/n$.

The second derivative cannot be represented in the straight forward way since the second derivative of the shape functions is identical zero everywhere. But actually we do not need it directly because we are to evaluate certain integrals, where they occur in the integrand. Evaluation of the second derivative of the shape functions can be avoided by the application of integration-by-parts technique:

$$\int_a^b u(x) \cdot v'(x)\ dx = [u(x) \cdot v(x)]_a^b - \int_a^b u'(x) \cdot v(x)\ dx$$

a.) Formulate the Galerkin finite-element method for this problem. Make sure the given boundary conditions are included.

b.) Write down the resulting system of algebraic equations of the shape

$$\mathbf{K}\vec{u} = \vec{f}.$$

Deduce explicitly the entries $k_{i,j}$ of $\mathbf{K}$ and the components of $\vec{f}$.