Tutorial

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Time of tutorial: Monday, 9:00 - 9:45 am

Overview

Matlab

Finite Difference Discretization of the Poisson-Equation
1.1. Matlab

- All exercises in Matlab
- gain understanding of mathematical methods / numerical algorithms (i.e. no Matlab programming course)

- Rechnerhalle / lxhalle
  - login to lxhalle.informatik.tu-muenchen.de
  - $ /mount/applic/packages/matlab/bin/matlab

- for free: Floating Licenses for TUM students (see Webpage)

- Octave / QtOctave also ok
- available in most linux distributions prepackaged
Matlab - Getting help

- Online resources:

- Matlab built-in help: type `help` or `help KEYWORD`
1.2. Finite Difference Discretization of the Poisson-Equation

We want to solve the **Poisson-Equation**:

\[ \Delta u(x) = f(x), \quad x \in \Omega, \quad \Omega \subseteq \mathbb{R}^n \]

with **Dirichlet-Boundary-Conditions**

\[ u(x) = 0, \quad x \in \partial \Omega \]

The **Laplacian operator** is defined as

\[ \Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \ldots + \frac{\partial^2}{\partial x_n^2} \]

Solve 1-d Poisson-Equation on unit intervall:

\[ \frac{\partial^2}{\partial x^2} u(x) = 0, \quad x \in ]0; 1[, \quad u(0) = 0, \quad u(1) = 0; \]

Usually, solutions of PDEs can’t be determined analytically

⇒ calculate numerically for certain values.
Finite Difference Discretization

Deduced from Taylor-Series in point $x$

(I) $u(x_k + h) = u_{k+1} = u(x_k) + hu'(x_k) + \frac{h^2}{2}u''(x_k) + O(h^3)$

(II) $u(x_k - h) = u_{k-1} = u(x_k) - hu'(x_k) + \frac{h^2}{2}u''(x_k) - O(h^3)$

(I) – (II), solve by $u'(x_k)$:

$$u'(x_k) = \frac{u(x_{k+1}) - u(x_{k-1})}{2h}$$

(I) + (II), solve by $u''(x_k)$:

$$u''(x_k) = \frac{u(x_{k+1}) - 2u(x_k) + u(x_{k-1})}{h^2}$$
Solve 1-d Poisson-Equation on unit interval:

\[ \frac{\partial^2 u(x)}{\partial x^2} = 0, \quad x \in ]0;1[, \quad u(0) = 0, \quad u(1) = 0; \]

Approximation: \( u''(x_k) = \frac{u(x_{k-1}) - 2u(x_k) + u(x_{k+1})}{h^2} \)

k=1: \( u''(x_1) = \frac{u(x_0) - 2u(x_1) + u(x_2)}{h^2} = b_1 \)

k=2: \( u''(x_2) = \frac{u(x_1) - 2u(x_2) + u(x_3)}{h^2} = b_2 \)

k=3: \( u''(x_3) = \frac{u(x_2) - 2u(x_3) + u(x_4)}{h^2} = b_3 \)