

Enhanced divergence-free elements for efficient incompressible flow simulations in the PDE framework Peano

Tobias Neckel,

Miriam Mehl, Christoph Zenger

Scientific Computing in Computer Science,
Fakultät für Informatik
TU München
Germany

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Outline

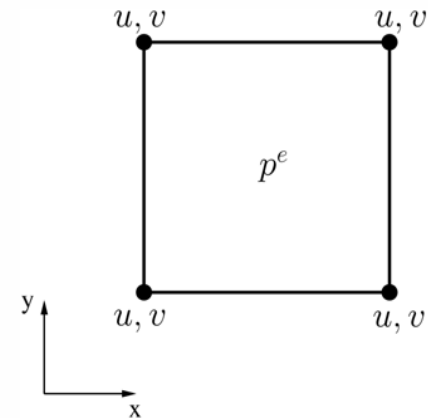
- Derivation of Div-free Ansatz Functions
- Enhanced Div-free Elements
- Numerical Results
 - Checkerboard Driven Cavity
 - Driven Cavity
 - Flow around a Cylinder
- Outlook

Derivation of Div-free Ansatz Functions

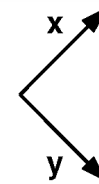
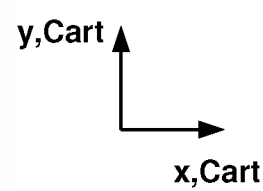
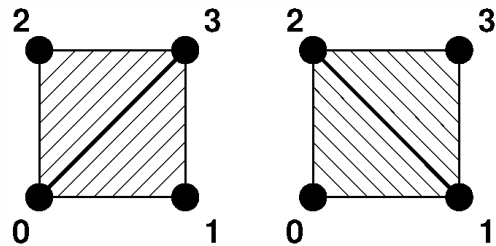
- Incompressible Navier-Stokes Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \nabla p - \nu \Delta \mathbf{u} = 0 \quad \nabla \cdot \mathbf{u} = 0$$

- Discretisation
 - low-order FEM (Q1Q0, etc.)
- Divergence-free elements:
 - solenoidal velocities in **every** point in a cell
 - simultaneous conservation of momentum **AND** energy

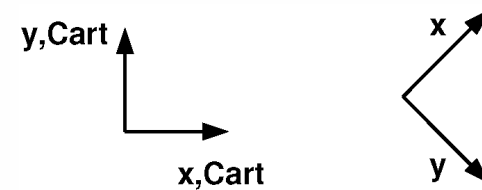
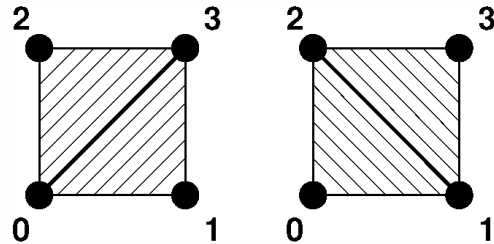


Derivation of Div-free Ansatz Functions



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Derivation of Div-free Ansatz Functions



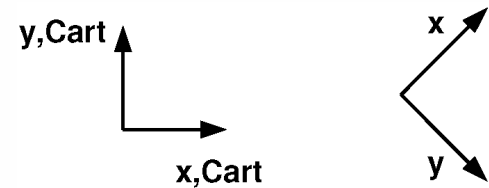
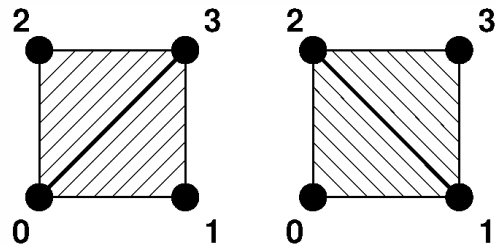
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$$du/dx = \text{const} = u_3 - u_0$$

$$dv/dy = \text{const} = v_1 - v_2$$

$$\text{div}(u) = u_3 - u_0 + v_1 - v_2 = \text{const} = 0$$

Derivation of Div-free Ansatz Functions

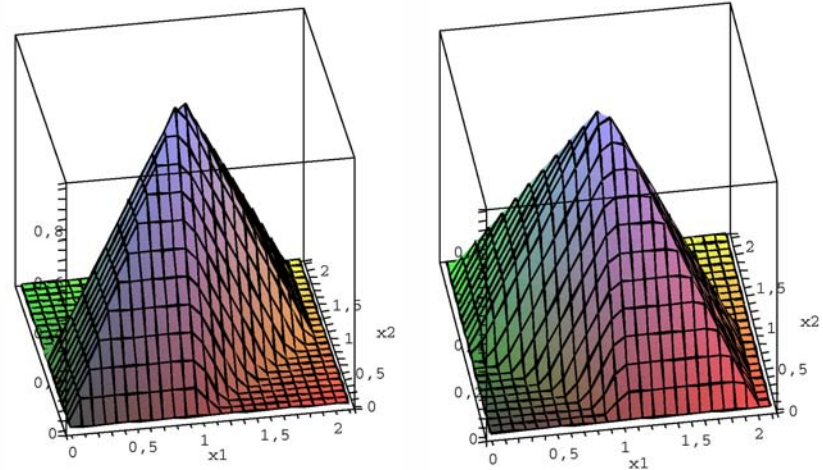


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$$du/dx = \text{const} = u_3 - u_0$$

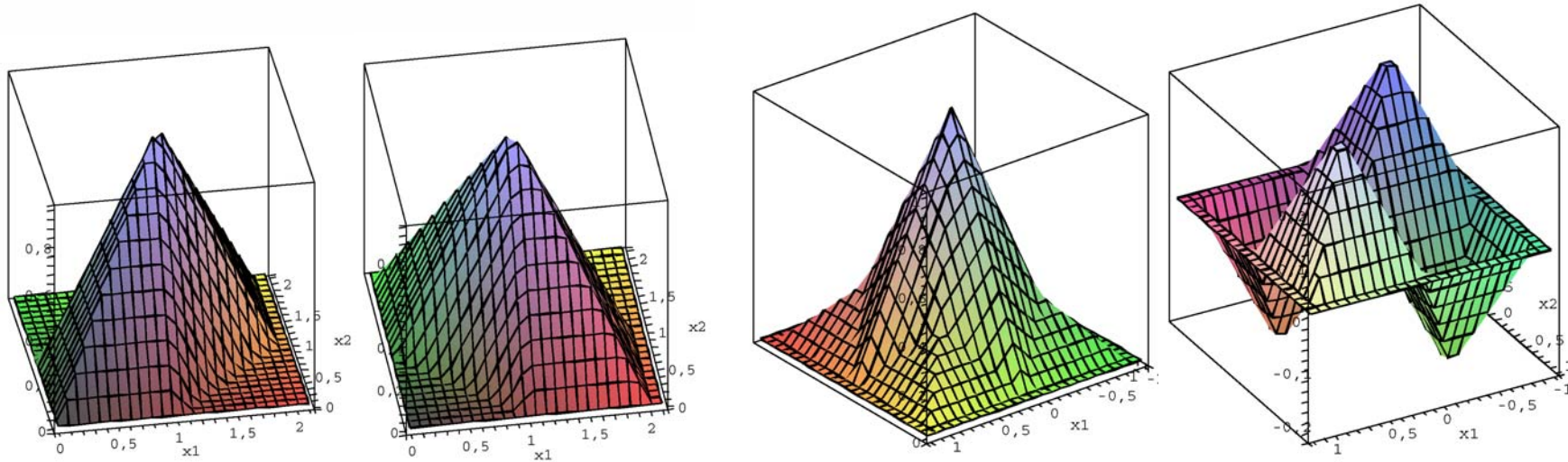
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Derivation of Div-free Ansatz Functions

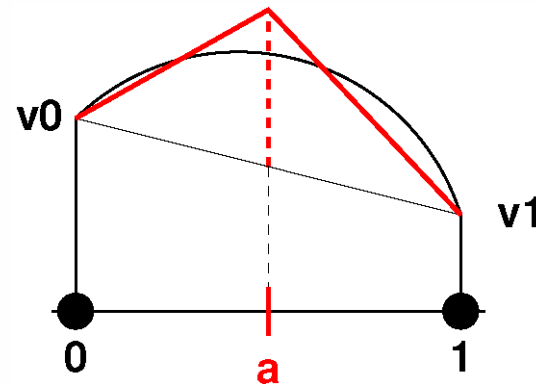
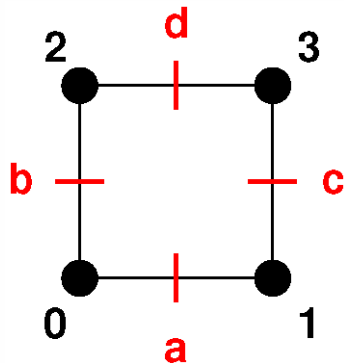
- Ansatz functions 45 and Cartesian:



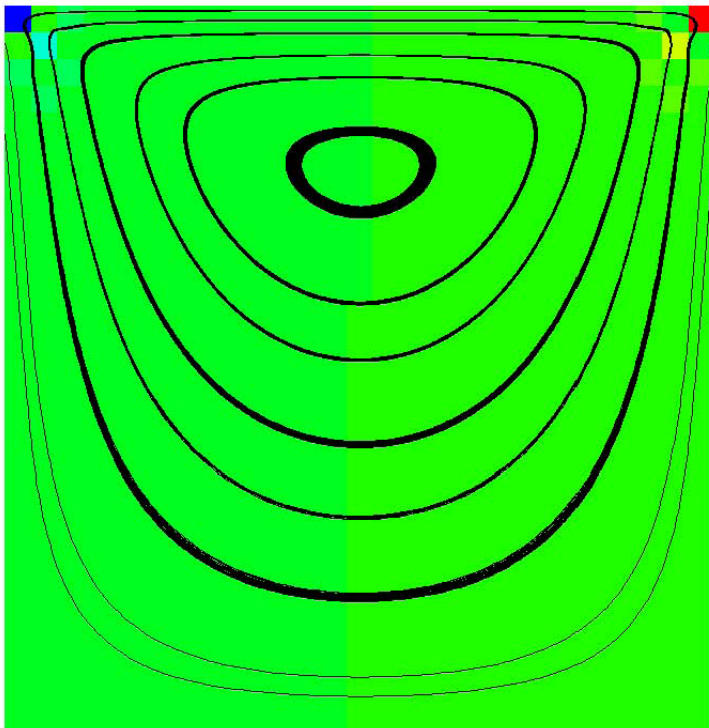
- Advantages of 45 representation:
 - Simplified derivation and representation of elements
 - Performance: ~20% less runtime for evaluation of operators D and C

Enhanced Div-free Elements

- Additional DoF on faces:
 - exact representation of fluxes on edges
 - no checkerboarding

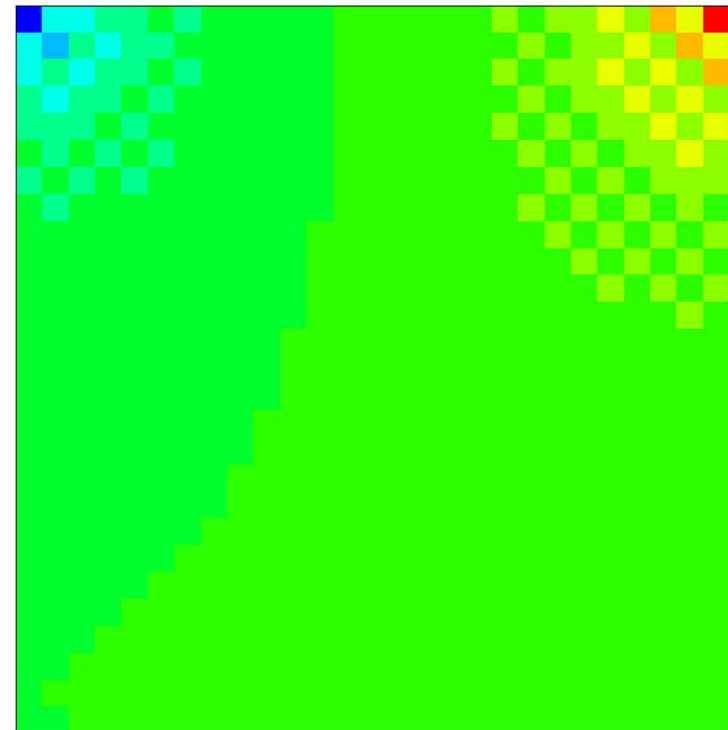


Numerical Results – Checkerboard Driven Cavity



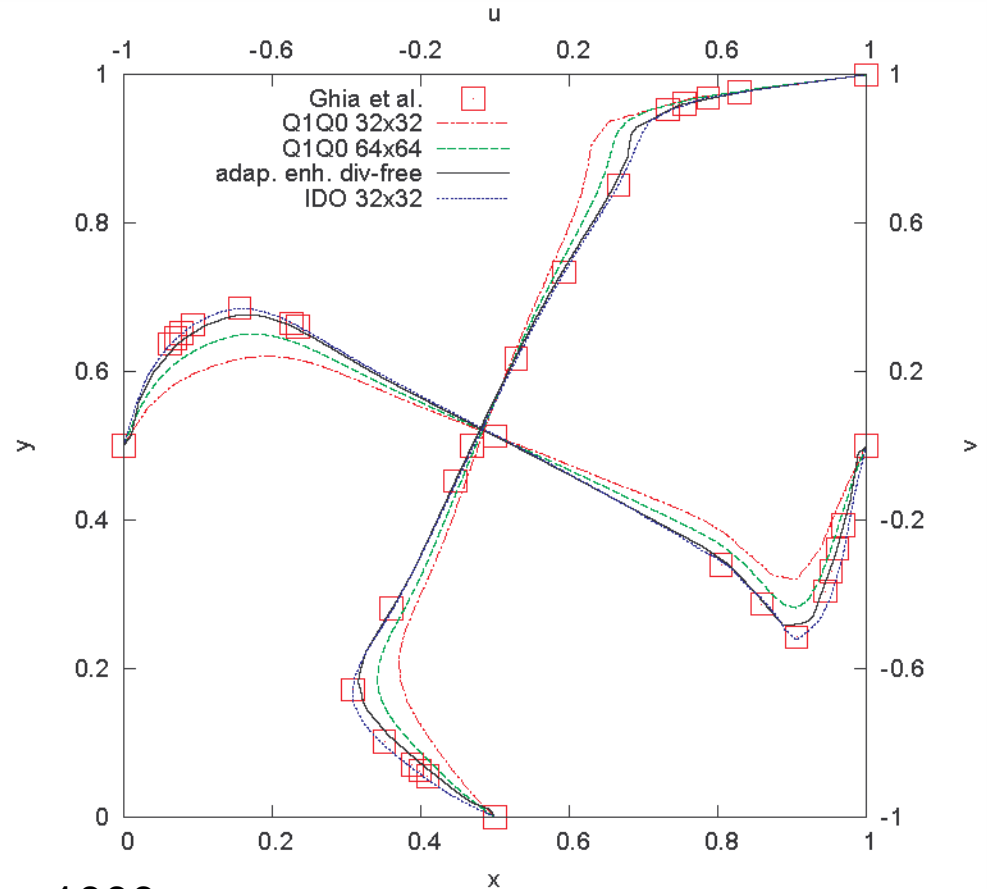
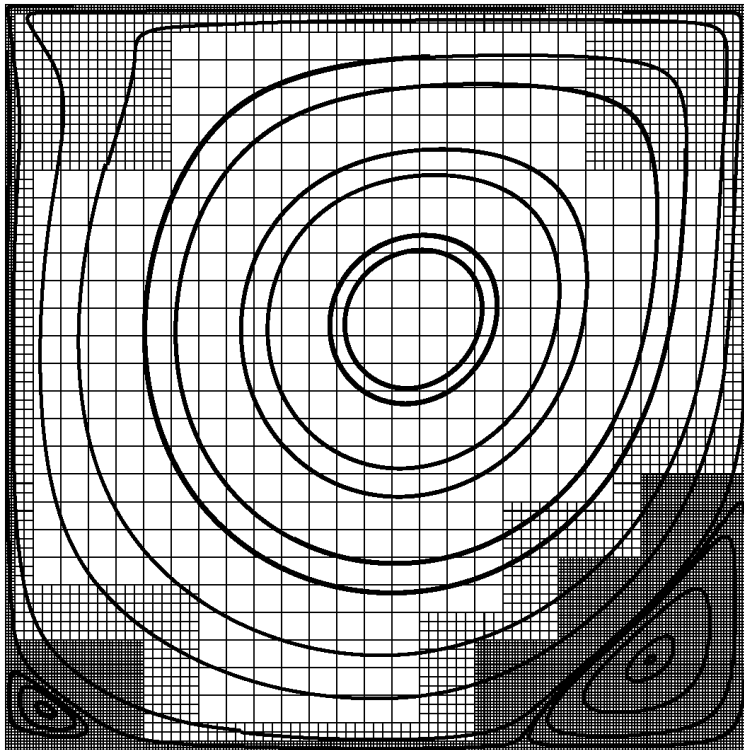
enhanced div-free
(steady state)

$Re=1$



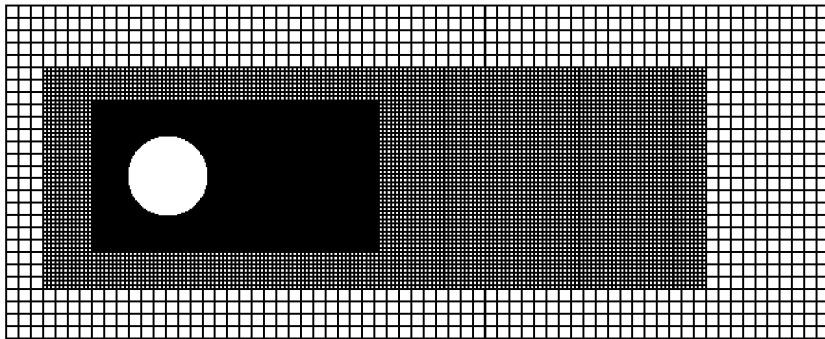
Q1Q0
(step1, **no convergence!**)

Numerical Results – Driven Cavity

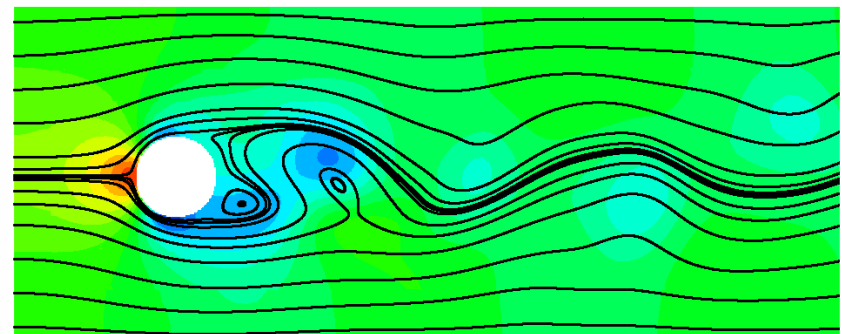
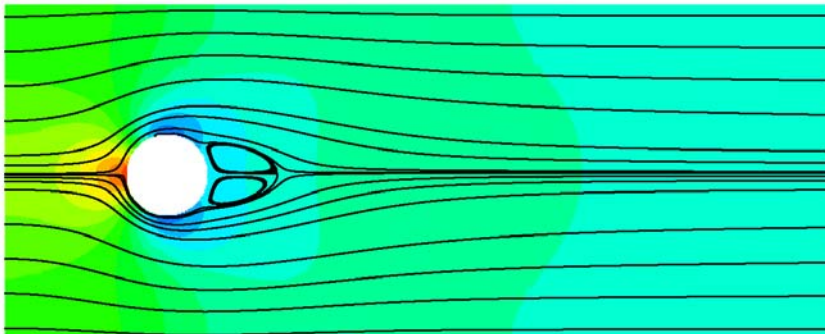


Re=1000

Numerical Results – Flow around a Cylinder



# DoF	Re = 20		Re = 100		
	C_d	C_l	$C_{d,max}$	$C_{l,max}$	St
88,857	5.68	0.0151	3.225	0.94	0.299
ref.	5.58	0.0107	3.230	1.00	0.298



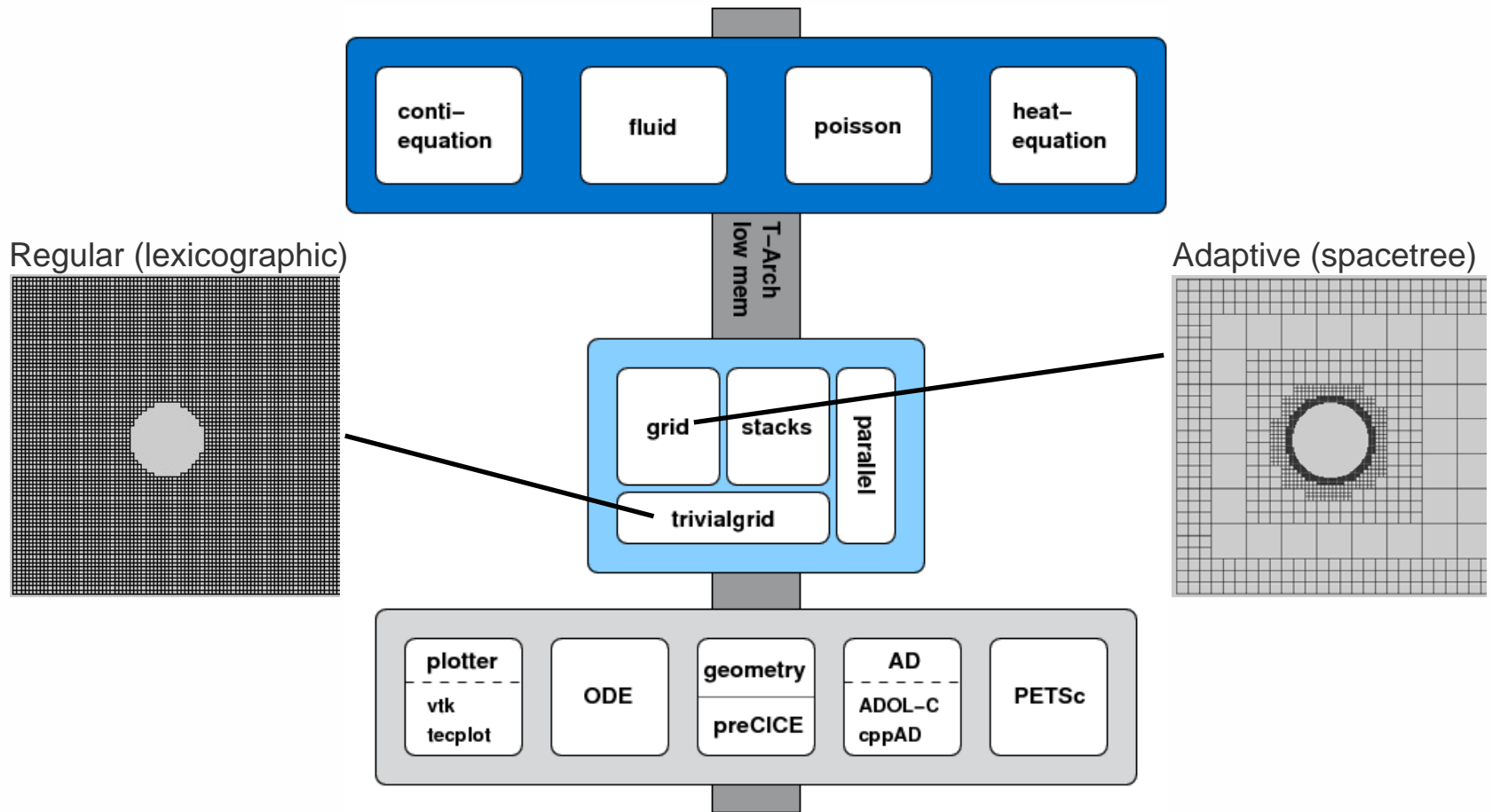
Outlook

- **Multigrid**
 - Peano Framework designed for hierarchical applications
 - Speed up computations while keeping low memory requirements
- Extension of (enhanced) div-free elements to **3D**

Thanks for your attention!



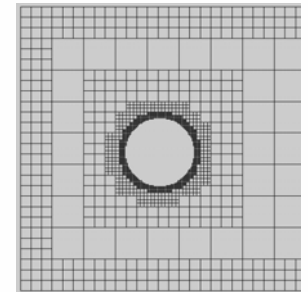
The PDE Framework Peano



<http://www5.in.tum.de/peano/>

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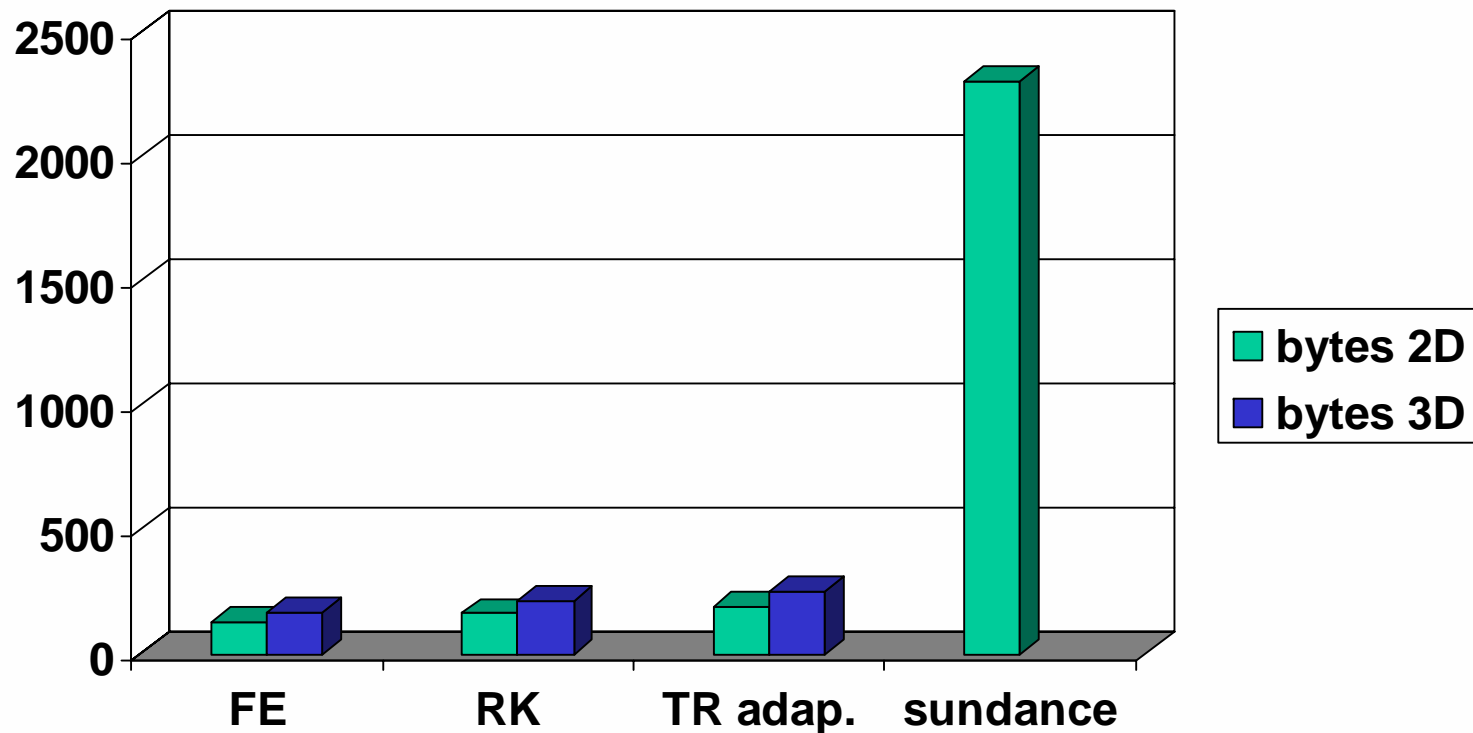
- Cartesian grids (arbitrary dimensions)
- Plug-in concept for applications
- Space-filling curves, spacetrees, and stack data structures
 - Strictly element-wise access
 - Low memory demands
 - Dynamical load balancing
 - Moving geometries, dynamical adaptivity, geometric multigrid
- Software Engineering
 - automatic tests, continuous integration, OO, design patterns, ...
- CFD component
 - Incompressible flow (FEM, IDO)
 - Explicit + implicit time-integration schemes (FE, RK4, BE, (adaptive) TR)



<http://www5.in.tum.de/peano/>

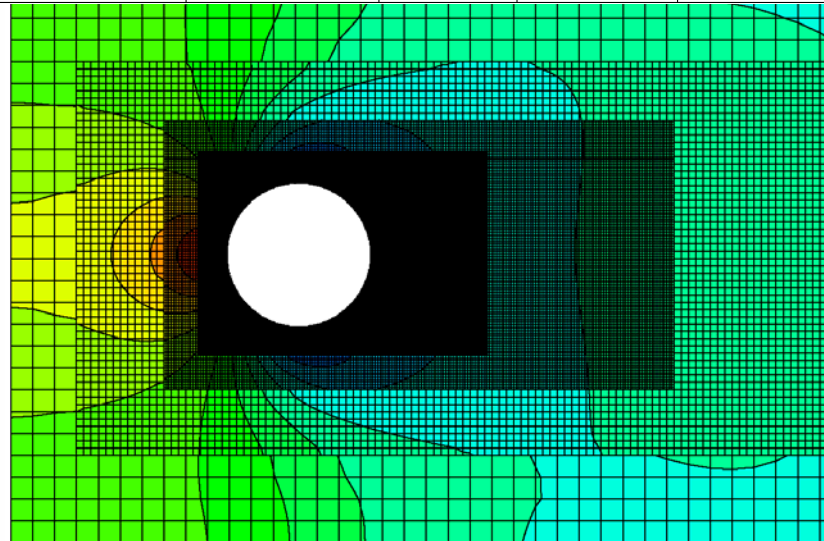
Backup I

Low memory requirements (FEM + adap.):

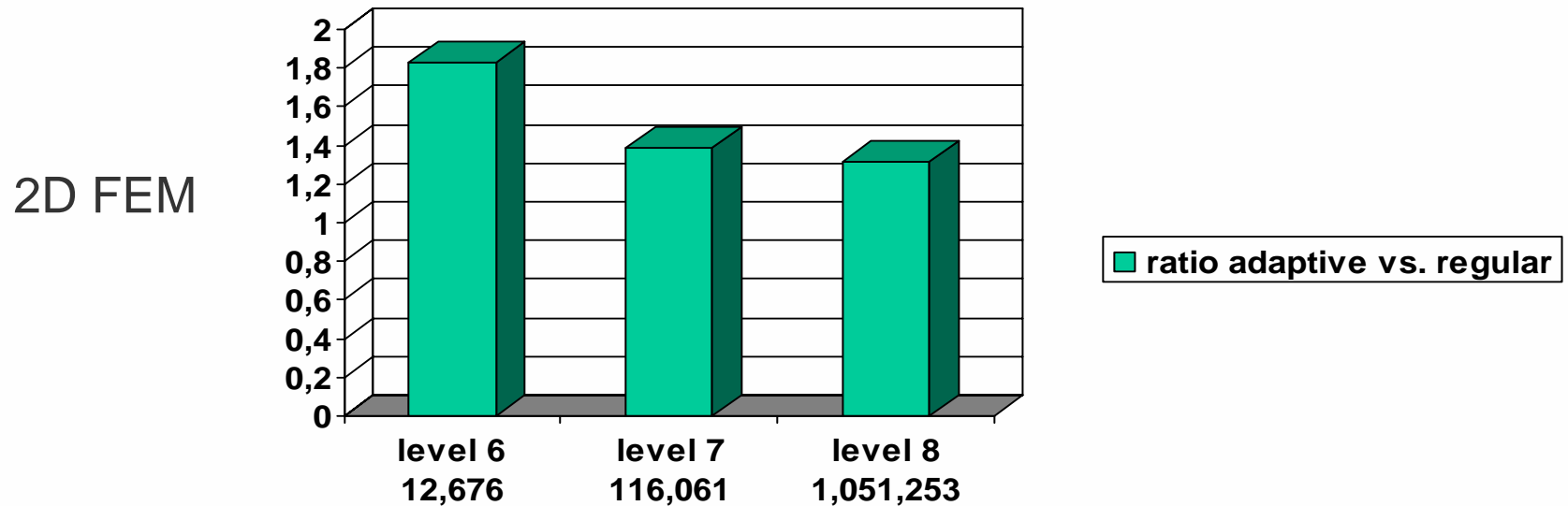


Numerical Results – FEM Q1Q0

max. level	min. level	#total DoF	c_d	c_l	CPU time per time step
8	6 (box)	88857	5.680	0.0150	0.71
9	7	125041	5.591	0.0113	1.03
9	8	1057877	5.561	0.0112	9.46
9	6 (box)	261501	5.586	0.0115	2.46
ref. data		–	5.580	0.0107	–



Numerical Results - Performance



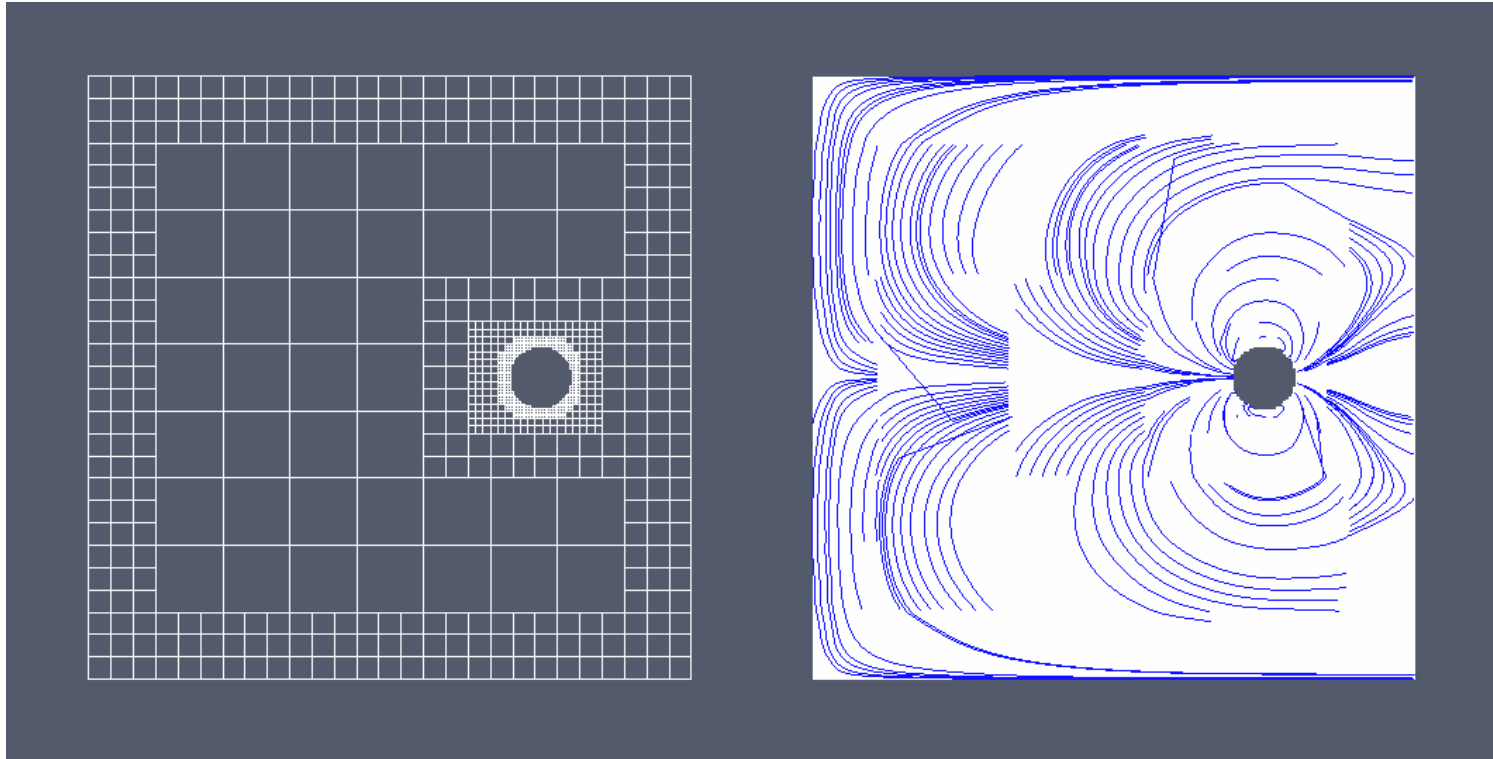
3D FEM

Overhead adaptive vs. regular < 3%

2D IDO

Overhead Peano vs. Aoki (regular): 1.3 – 4.4

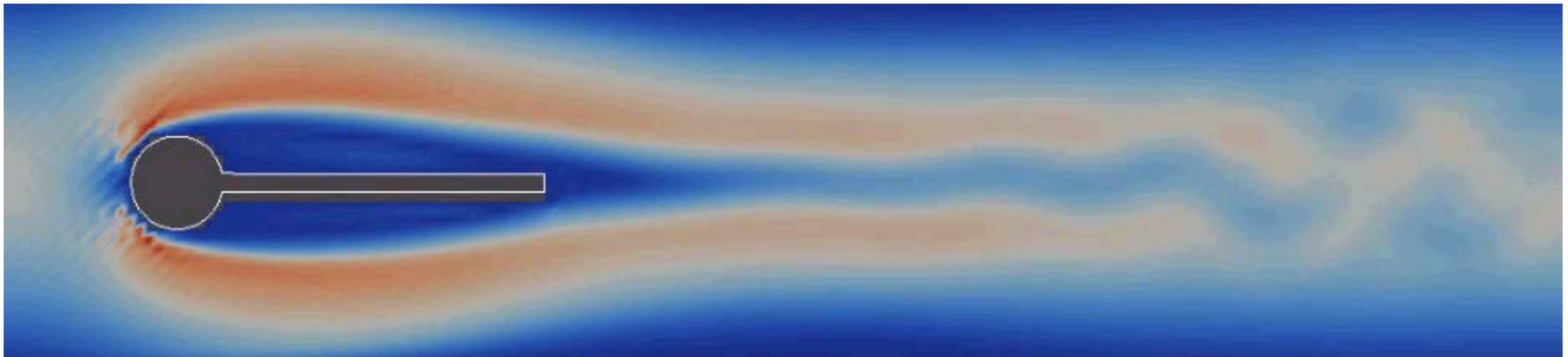
CFD Extensions



- Moving geometries
 - Update of data + grid (regular + adaptive)
 - Divergence correction

joint work with Kristof Unterwiesing

CFD Extensions



joint work with Janos Benk, Bernhard Gatzhammer, Miriam Mehl, Kristof Unterweger, and Tobias Weinzierl

- Moving geometries
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