Spatial Discretisation Schemes in the PDE framework Peano for Fluid-Structure Interactions

T. Neckel, H.-J. Bungartz, B. Gatzhammer, M. Mehl, C. Zenger

TUM
Department of Informatics
Chair of Scientific Computing

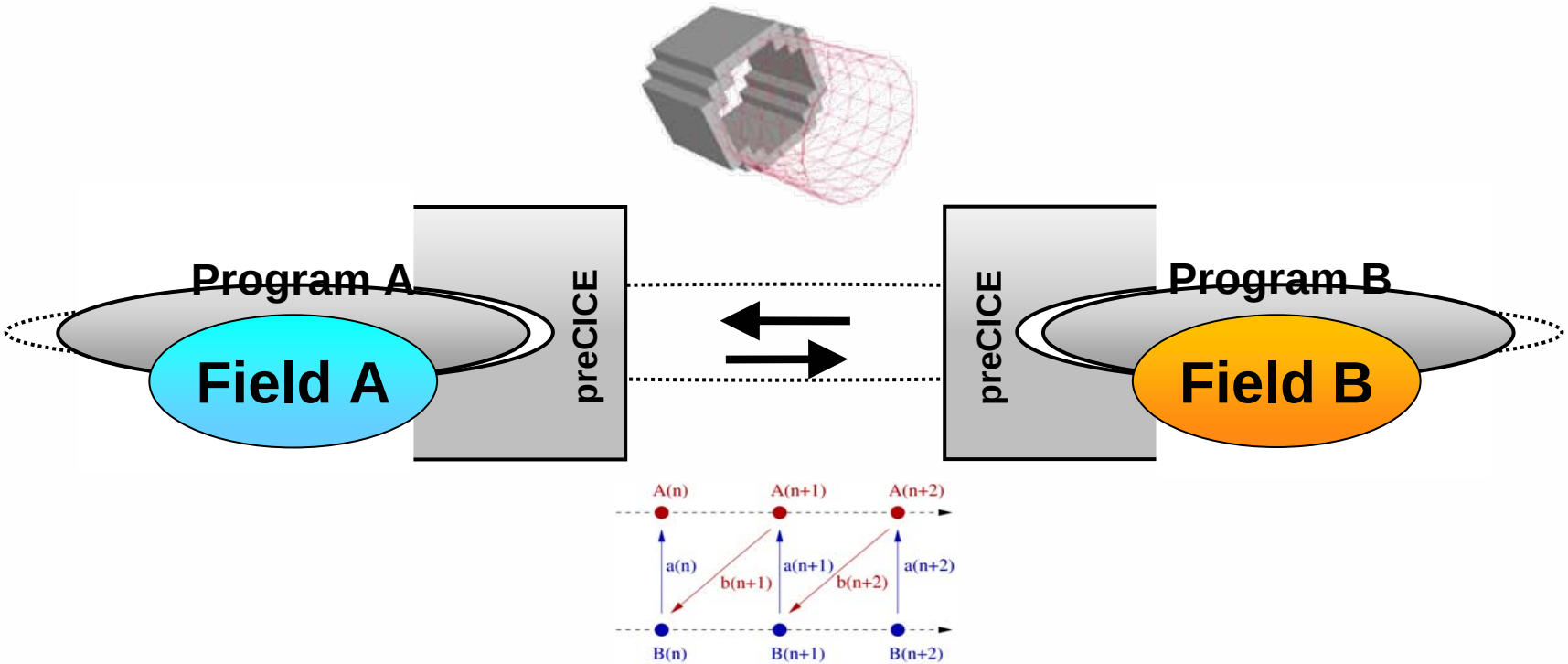
IMECE 2010, Vancouver, November 16, 2010

Outline

- Introduction
  - our FSI approach
- Div-free Elements
  - Derivation
  - Driven Cavity
  - Flow around a Cylinder
- Enhanced Div-free Elements
- Outlook

Partitioned FSI using preCICE

- no physical central control unit, local preCICE controllers instead
- P2P solver communication



preCICE – Features

Coupling schemes:
- Staggered explicit
- Implicit with
  - constant under-relaxation
  - Aitken- based under-relaxation
  - IQN-ILS acceleration
- Convergence measurements
- Sub-cycling
- Checkpointing

Conservative/consistent linear projections data mapping

Coupled solvers:
- CFD
  - Peano (in-house)
  - Fluent (commercial)
- CSD
  - AdhoC (academic, E. Rank et al.)
  - Structure0815 (simple in-house)
  - Comsol Multiphysics (commercial)

preCICE – Features

Coupling schemes:
- Staggered explicit
- Implicit with
  - constant under-relaxation
  - Aitken- based under-relaxation
  - **IQN-ILS acceleration**
- Convergence measurements
- Sub-cycling
- Checkpointing

Conservative/consistent linear projections data mapping

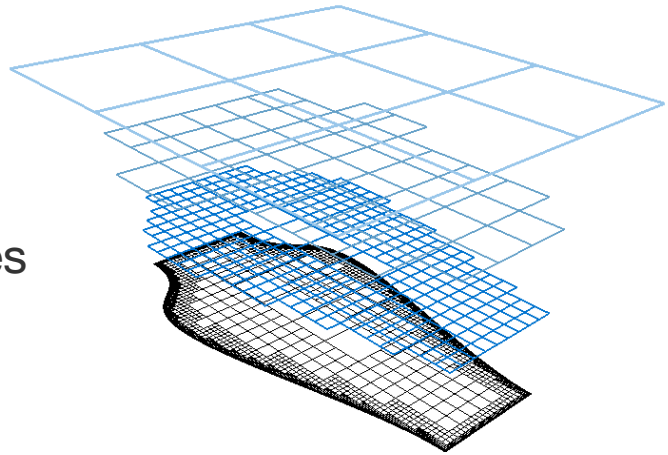
Coupled solvers:
- CFD
  - **Peano** (in-house)
  - Fluent (commercial)
- CSD
  - AdhoC (academic, E. Rank et al.)
  - Structure0815 (simple in-house)
  - **Comsol Multiphysics** (commercial)

The PDE Framework Peano

- Cartesian grids

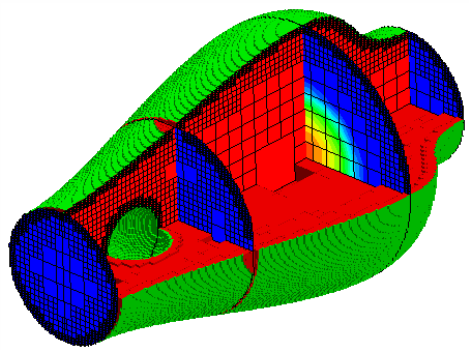
  (recursive adaptivity, full grid hierarchy)
- Low memory requirements
- Space-Filling curves + stack data structures

  ➔ high cache-hit rates (>98%)
- Shared/distributed mem. parallelisation



source: T. Weinzierl

- Software engineering aspects



Three simple FSI examples

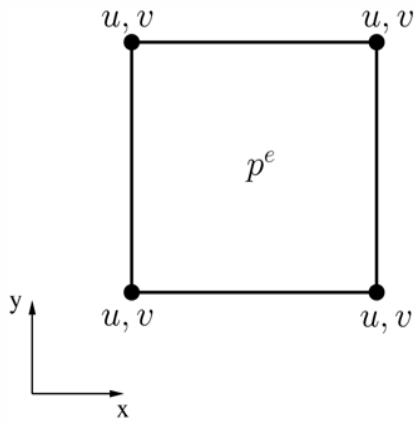
- Video 1: flexible sphere in channel flow
- Video 2: 2D cantilever benchmark

CFD – Spatial Discretisation

- Incompressible Navier-Stokes Equations

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0 \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} + \frac{1}{\rho}\nabla p - \nu\,\Delta \mathbf{u} &= 0 \end{aligned}$$

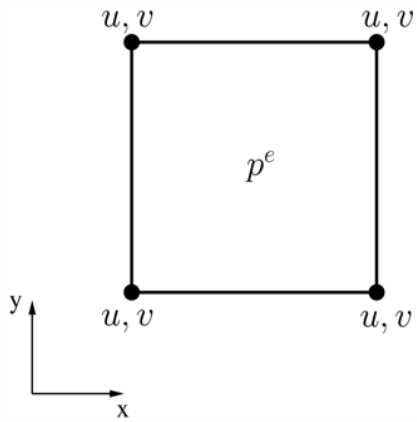
- Discretisation
  - low-order FEM (Q1Q0, etc.)



CFD – Spatial Discretisation

- Incompressible Navier-Stokes Equations

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0 \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} + \frac{1}{\rho}\nabla p - \nu\,\Delta \mathbf{u} &= 0 \end{aligned}$$



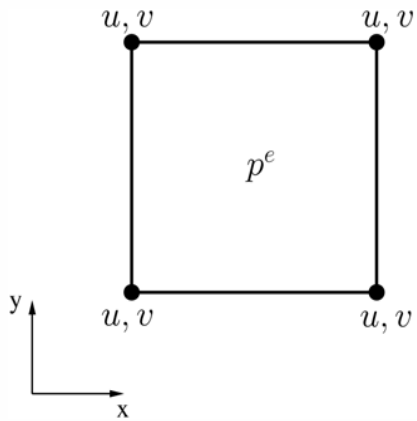
- Discretisation
  - low-order FEM (Q1Q0, etc.)

- Idea 1: enlarge elements to avoid CB modes
- Idea 2: divergence-free elements:
  - solenoidal velocity field in **every** point in a discrete cell
  - simultaneous conservation of momentum **AND** energy

CFD – Spatial Discretisation

- Incompressible Navier-Stokes Equations

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0 \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} + \frac{1}{\rho}\nabla p - \nu\,\Delta \mathbf{u} &= 0 \end{aligned}$$

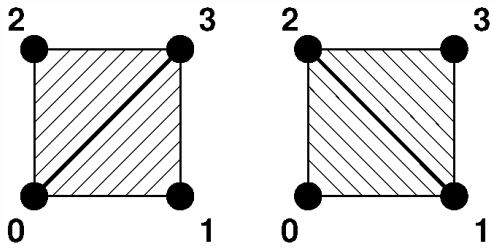


- Discretisation
  - low-order FEM (Q1Q0, etc.)

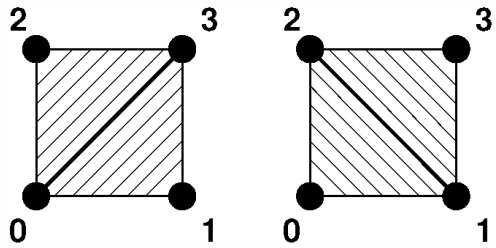
- Idea 1: enlarge elements to avoid CB modes
- Idea 2: divergence-free elements:
  - solenoidal velocity field in **every** point in a discrete cell
  - simultaneous conservation of momentum **AND** energy

} Combine

Div-free Elements – Derivation



Div-free Elements – Derivation

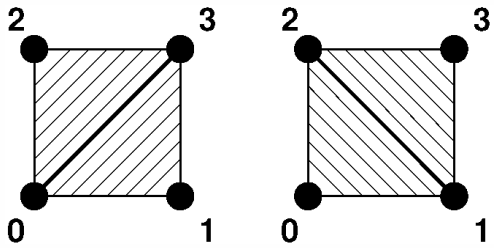


$du/dx = const = u3 - u0$

$dv/dy = const = v1 - v2$

$div(u) = u3 - u0 + v1 - v2 = const = 0$

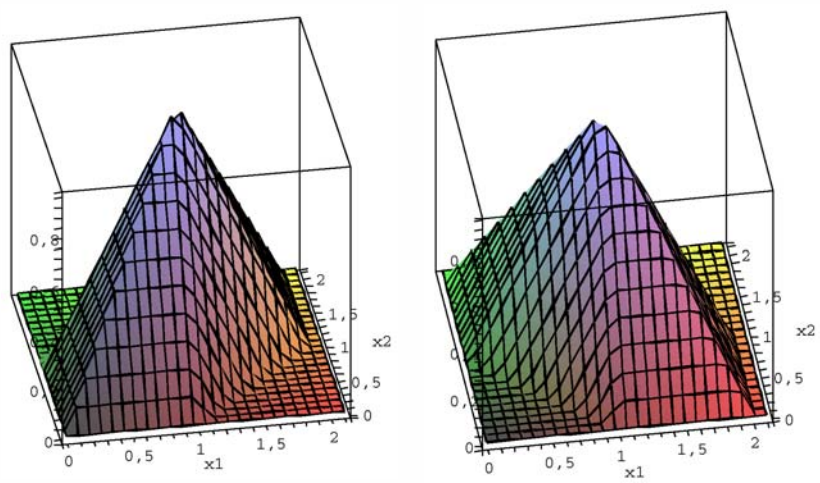
Div-free Elements – Derivation



du/dx = const = u3 - u0

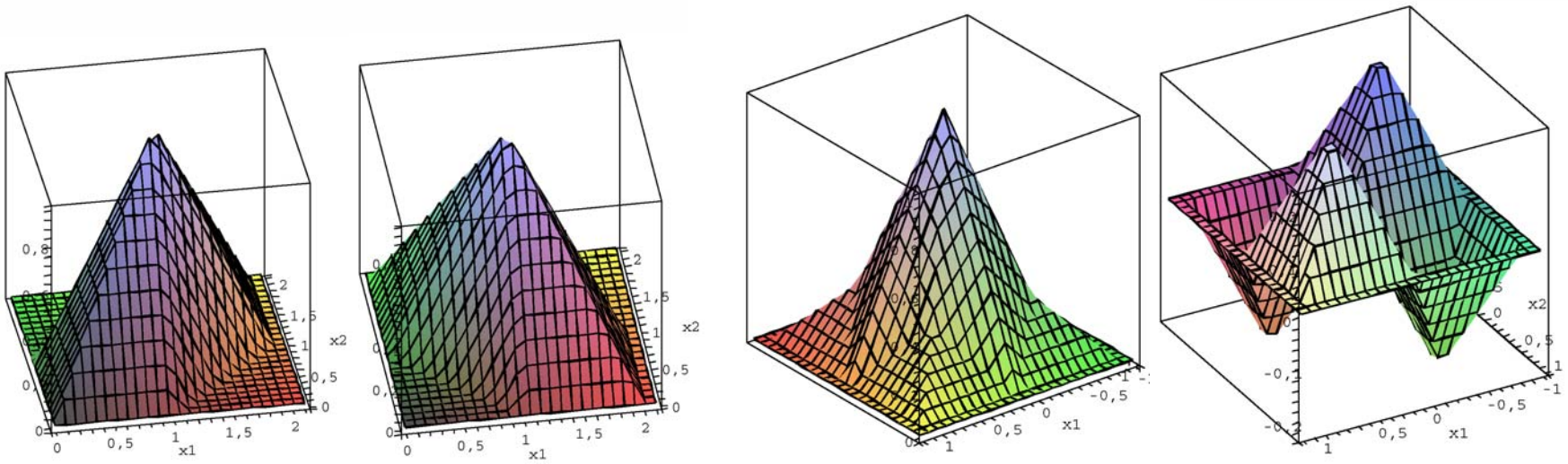
dv/dy = const = v1 - v2

div(u) = u3 - u0 + v1 - v2 = const = 0



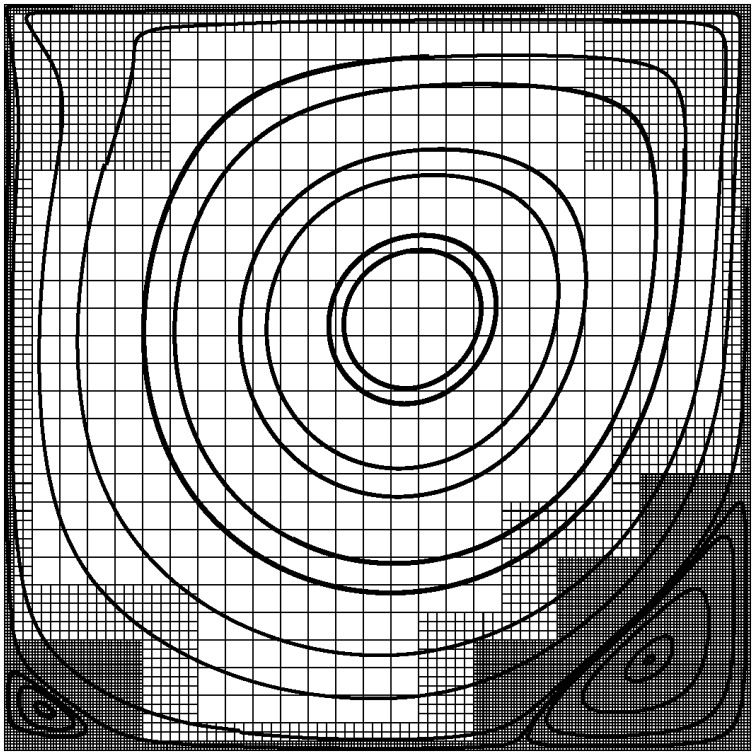
Div-free Elements – Derivation II

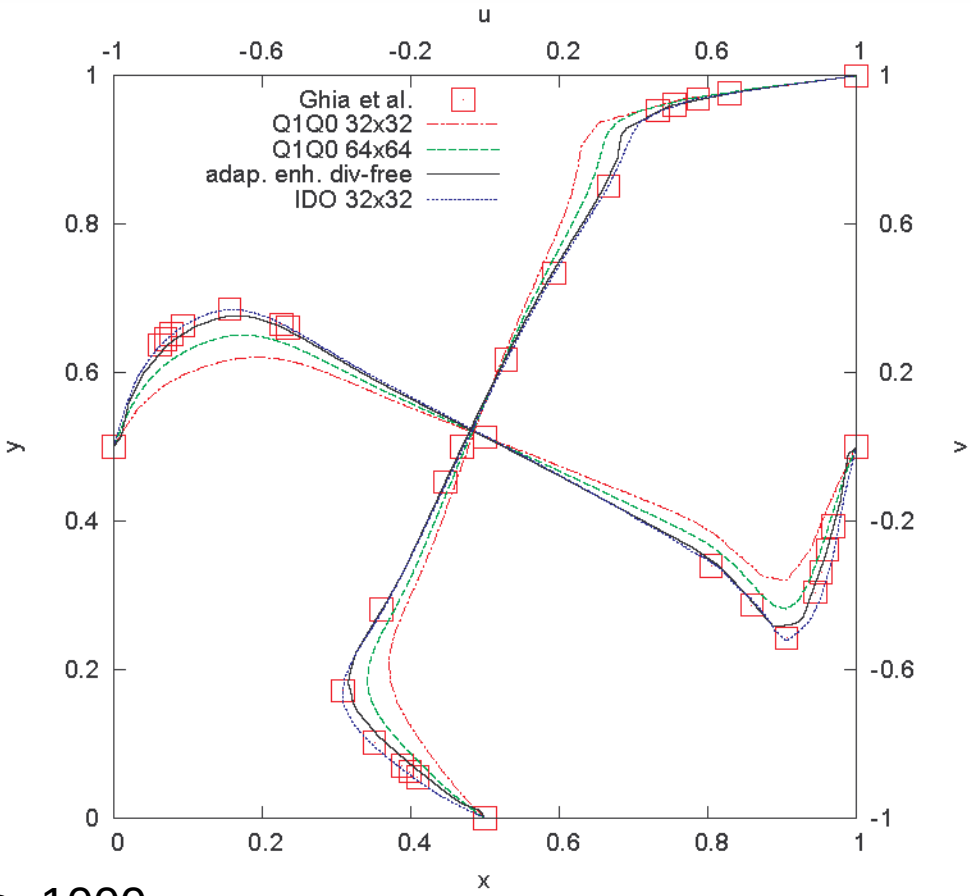
- Ansatz functions 45 and Cartesian:



- Advantages of 45 representation:
  - Simplified derivation and representation of elements
  - Performance: ~20% less runtime for evaluation of operators D and C

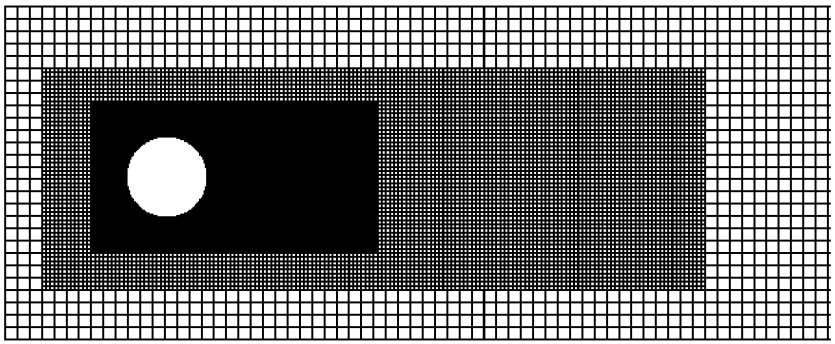
Div-free Elements – Driven Cavity



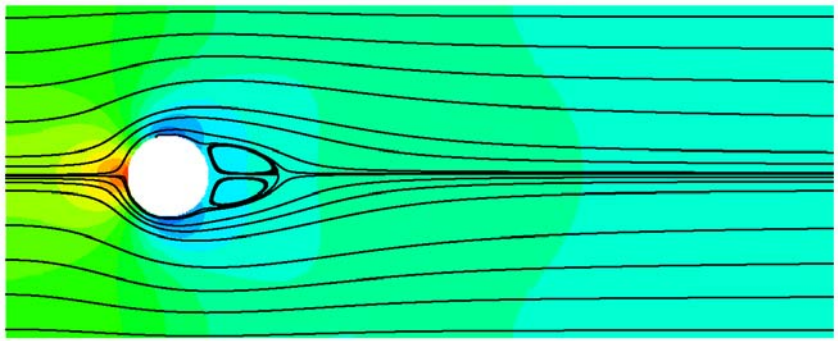


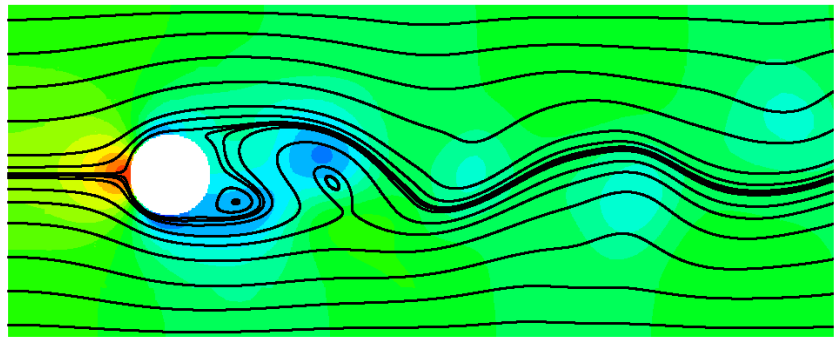
Re=1000

Div-free Elements – Flow around a Cylinder



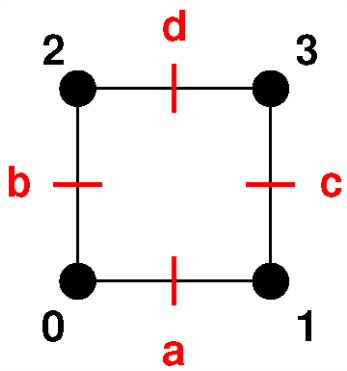
# DoF	Re = 20		Re = 100		
	c_d	c_l	$c_{d,max}$	$c_{l,max}$	St
88,857	5.68	0.0151	3.225	0.94	0.299
ref.	5.58	0.0107	3.230	1.00	0.298

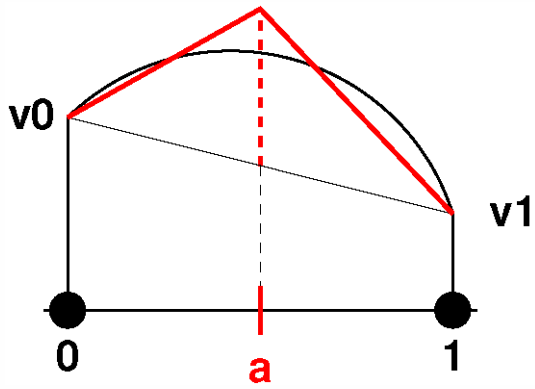




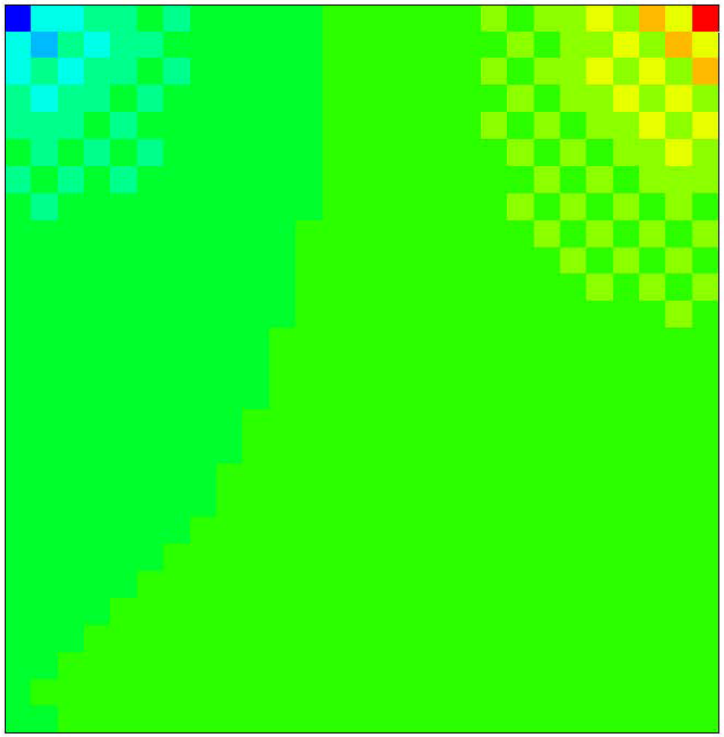
Enhanced Div-free Elements

- Additional DoF on faces:
  - exact representation of fluxes on edges
  - no checkerboarding



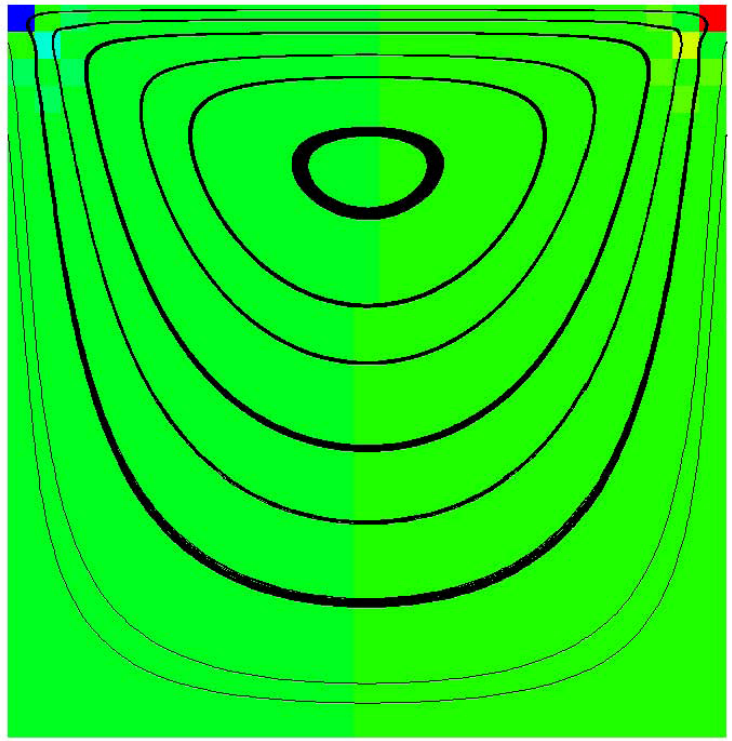


Enh. Div-free Elements – Checkerboard Driven Cavity



Q1Q0
(step1, no convergence!)

Re=1



enhanced div-free
(steady state)

Outlook

- Adaptive enhancement
- Multigrid
  - Peano Framework designed for hierarchical applications
  - Speed up computations while keeping low memory requirements
- Extension of (enhanced) div-free elements to 3D

Thanks for your attention!