Research Stay Abroad at Aoki Laboratory

# Achievements at Aoki Laboratory: Parallelization of the OU Process, Runs on TSUBAME

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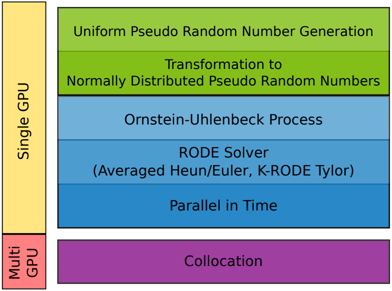
# Overview

## What has been planned



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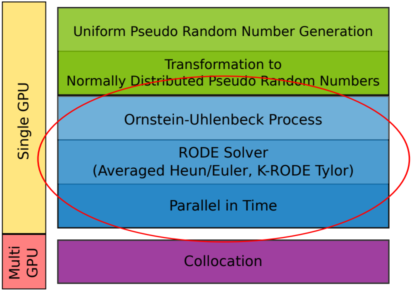

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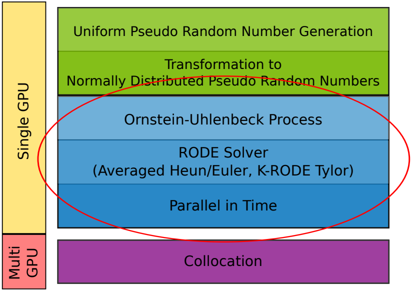


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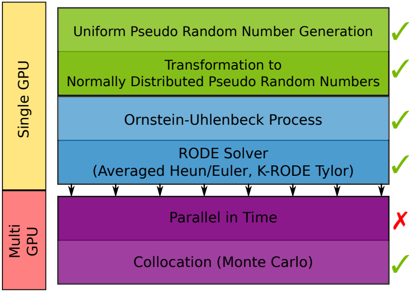

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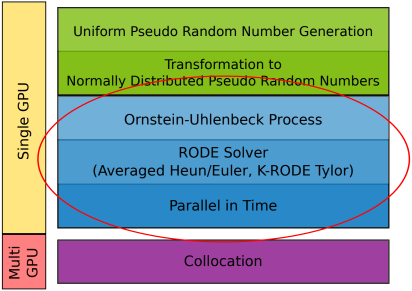


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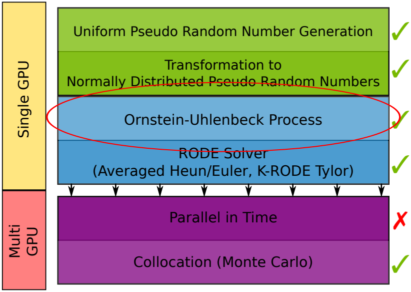

# How to parallelize the Ornstein-Uhlenbeck process

# A closer look on the solution of the OU process (1/2)

## The Ornstein-Uhlenbeck (OU) process

$$\mathrm{d}O_t = -O_t\mathrm{d}t + \mathrm{d}W_t$$

$O_t$: Ornstein-Uhlenbeck process
$W_t$: Wiener process (e.g. white noise)

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## Solution of the OU process

$$O_{t+h} \quad = \quad \mu O_t + \sigma_X n_1$$

$h$: Time step size
$O_{t+h}$: Realization of the OU process at the next point of time
$\mu$: $\mu = e^{-h}$
$\sigma_X$: $\sigma_X = \sqrt{\frac{1-\mu^2}{2}}$
$n_1$: Sample value of a normally distributed random variable $\mathcal{N}(0, 1)$

# A closer look on the solution of the OU process (2/2)

## Some iterations of the solution

$$
\begin{aligned}
O_{t+h} &= \mu O_t + \sigma_X n_1^{(1)} \\
O_{t+2h} &= \mu O_{t+h} + \sigma_X n_1^{(2)} = \mu \left( \mu O_t + \sigma_X n_1^{(1)} \right) + \sigma_X n_1^{(2)} = \\
&= \mu^2 O_t + \sigma_X \left( \mu n_1^{(1)} + n_1^{(2)} \right) \\
O_{t+3h} &= \mu O_{t+2h} + \sigma_X n_1^{(3)} = \mu \left( \mu O_{t+h} + \sigma_X n_1^{(2)} \right) + \sigma_X n_1^{(3)} = \\
&= \mu \left( \mu \left( \mu O_t + \sigma_X n_1^{(1)} \right) + \sigma_X n_1^{(2)} \right) + \sigma_X n_1^{(3)} = \\
&= \mu^3 O_t + \sigma_X \left( \mu^2 n_1^{(1)} + \mu n_1^{(2)} + n_1^{(3)} \right) \\
\ldots &= \ldots \\
O_{t+kh} &= \mu^k O_t + \sigma_X \sum_{i=1}^{k} \left( \mu^{k-i} n_1^{(i)} \right)
\end{aligned}
$$

# Prefix Sum/Scan

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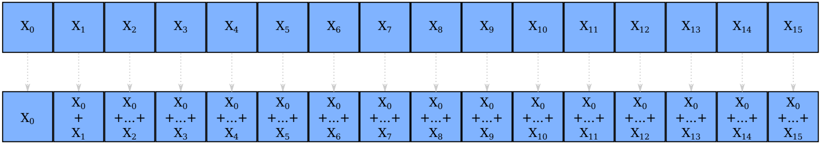
$$A_k = \sum_{i=0}^{k} X_i$$



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| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $X_0$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | $X_9$ | $X_{10}$ | $X_{11}$ | $X_{12}$ | $X_{13}$ | $X_{14}$ | $X_{15}$ |
| $X_0$ | $X_0 + X_1$ | $X_0 +\ldots+ X_2$ | $X_0 +\ldots+ X_3$ | $X_0 +\ldots+ X_4$ | $X_0 +\ldots+ X_5$ | $X_0 +\ldots+ X_6$ | $X_0 +\ldots+ X_7$ | $X_0 +\ldots+ X_8$ | $X_0 +\ldots+ X_9$ | $X_0 +\ldots+ X_{10}$ | $X_0 +\ldots+ X_{11}$ | $X_0 +\ldots+ X_{12}$ | $X_0 +\ldots+ X_{13}$ | $X_0 +\ldots+ X_{14}$ | $X_0 +\ldots+ X_{15}$ |

## Modified Prefix Sum/Modified Scan

$$A_k = \sum_{i=0}^{k} \left(\mu^{k-i} X_i\right)$$

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $X_0$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | $X_9$ | $X_{10}$ | $X_{11}$ | $X_{12}$ | $X_{13}$ | $X_{14}$ | $X_{15}$ |
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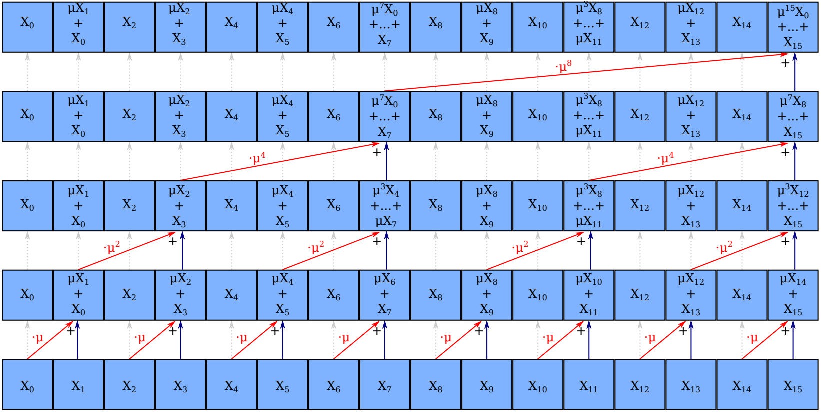
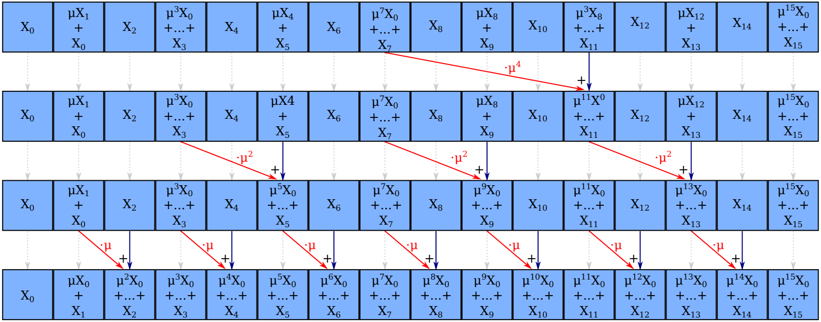
# Modified Parallel Scan for GPUs (1/2)

## Up Sweep



# Modified Parallel Scan for GPUs (2/2)

## Down Sweep



# Results on TSUBAME

# Computational Intensity of Stages

## Scenario

| | |
|---|---|
| **Model**: | Kanai/Tajimi earthquake model |
| **Solver**: | 3-RODE Taylor scheme |

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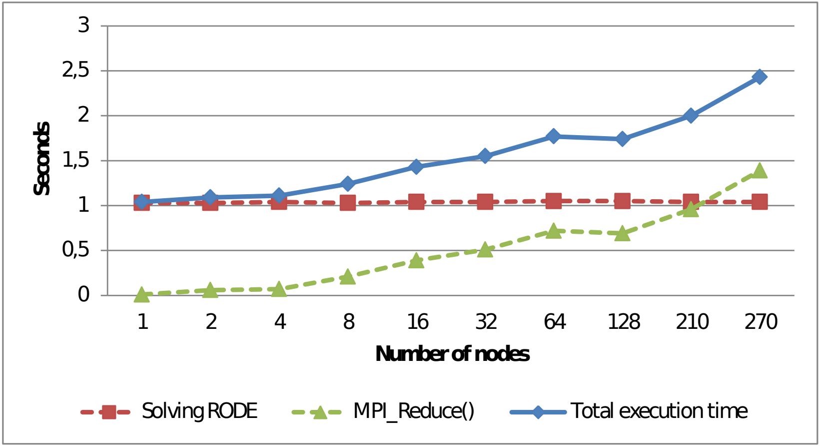
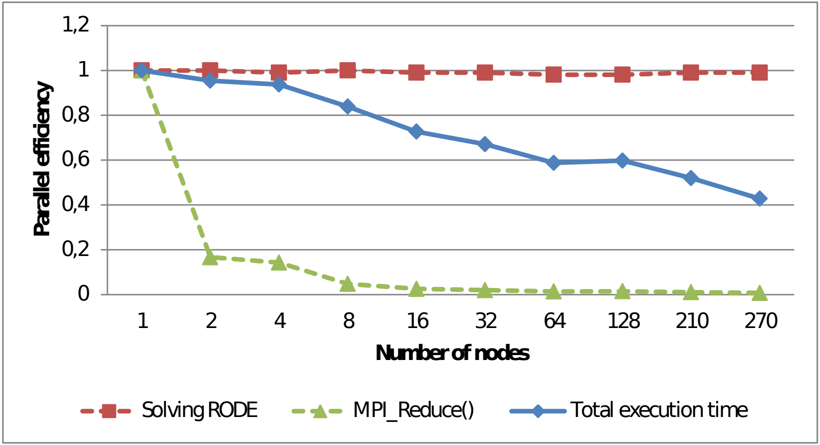
## Single GPU Performance

| Stage | Share | GFLOPS | % Peak | GB/s | % Peak |
|---|---|---|---|---|---|
| random numbers | 40.7% | 10.66 | 0.42% | 0.042 | 0.016% |
| OU process | 44.9% | $4.19 \cdot 10^{-3}$ | $1.06 \cdot 10^{-4}$% | 74.884 | 29.95% |
| averaging | 12.2% | 4.61 | 0.11% | 79.466 | 31.78% |
| solver | 2.2% | $0.96 \cdot 10^{-3}$ | $2.44 \cdot 10^{-5}$% | 0.120 | 0.048% |

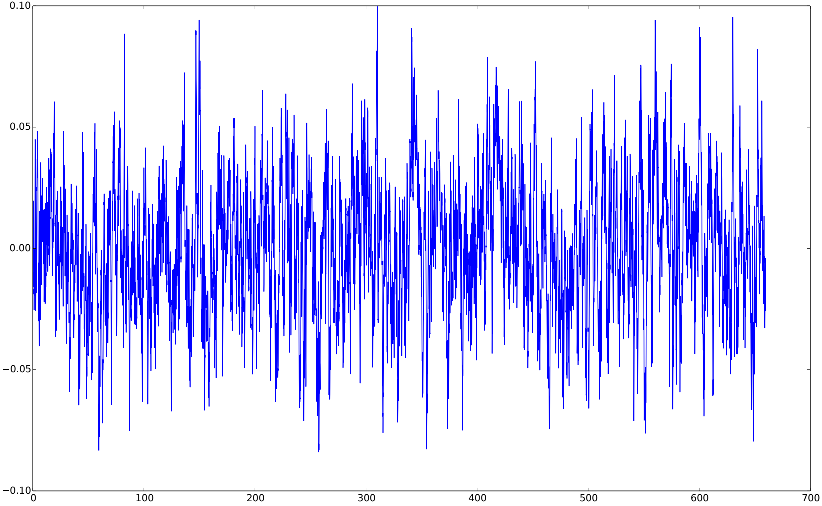
# Execution Time



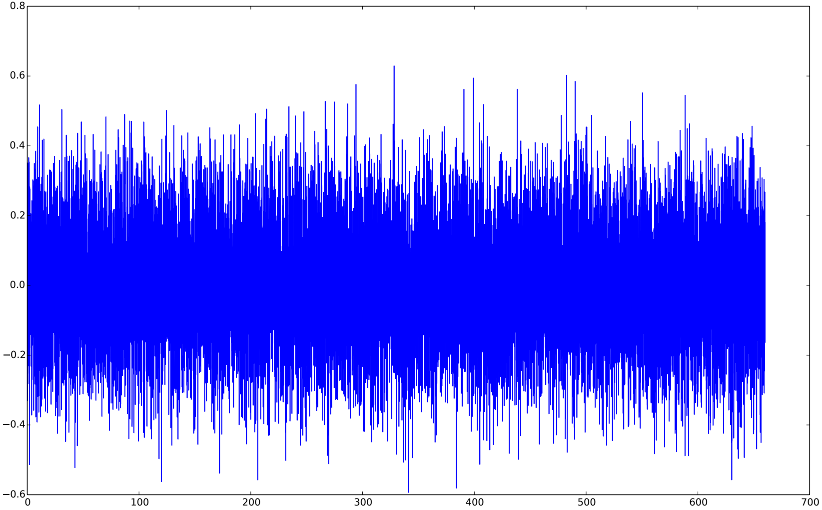
# Parallel Efficiency



# Earthquake: Ground Position



# Earthquake: Ground Acceleration



# Final slide