Error estimates in convergence acceleration process

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When a sequence (S_n) is converging too slowly to its limit S, it can often be transformed into a new sequence (T_n) converging, under some assumptions, faster to the same limit. In the first part of this lecture, it will be shown how convergence tests for series and estimates of the error $S_n - S$ can be used for constructing new sequence transformations for the acceleration of convergence. In the second part of the lecture, we will discuss how to control the error $T_n - S$ in the transformed sequence, which is a question of primary importance for stopping the process. In fact, intervals containing asymptotically the limit will be constructed.

[1]- C. Brezinski, M. Redivo-Zaglia, Extrapolation Methods. Theory and Practice, North-Holland, Amsterdam, 1991.

Error estimates in Padé approximation

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A Padé approximant is a rational function whose series expansion in ascending powers of the variable coincides with a given series as far as possible, that is up to the sum of the degrees of the numerator and the denominator inclusively. Such approximants are widely used for the approximation of functions in numerical analysis and in various branches of applied sciences.

After giving their definition, we will interpret them as formal Gaussian quadratures. It is well-known that the error in Gaussian quadratures can be estimated by using the Kronrod procedure. We will see that this procedure can be extended to Padé approximants thus providing a quite efficient way for estimating their error. The same procedure can be extended to the estimation of the error in some related convergence acceleration methods.

[1]- C. Brezinski, Error estimate in Padé approximation, in "Orthogonal Polynomials and their Applications", M. Alfaro et al. eds., Lecture Notes in Mathematics, vol. 1329, Springer Verlag, Heidelberg, 1988, pp. 1-19.

[2]- C. Brezinski, Procedures for estimating the error in Padé approximation, Math. Comput., 53 (1989) 639-648.

[3]- M. Bellalij, A simultaneous process for convergence acceleration and error control, J. Comput. Appl. Math., 33 (1990) 217-231.

Error estimates in the solution of systems of linear equation

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When solving a system of linear equations $Ax = b$ by a direct or an iterative method, an approximation y or a sequence of such approximations are obtained. In most cases, the quality of the approximate solution is judged by computing the residual $r = b - Ay$. However, in many cases, the residual can be small and the error $y - x$ large. We will show how to estimate the error by a very simple and efficient technique. The estimates will be justified by an analysis based on the singular value decomposition.

[1]- C. Brezinski, Error estimates in the solution of linear systems, submitted.