Interdisciplinary Guided Research at the Edge of Dynamical Systems & Scientific Computing

Ernst Otto Fischer-Lehrpreis der TUM

Ernst Otto Fischer-Lehrpreis 2011
der TU München

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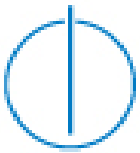


Tobias Neckel
Florian Rupp

Executive Summary

Interdisciplinary Guided Research at the Edge of Dynamical Systems & Scientific Computing (short: Dynamical Systems & Scientific Computing) introduces the theory and numerics of randomly disturbed differential equations from the point of view of both Dynamical Systems and Scientific Computing. The course is designed as a seminar with a dedicated workshop part supplemented by exercises and some few lectures. It is open (and meant for) students of mathematics as well as informatics: This interesting combination of different backgrounds will allow for fruitful discussion and solution approaches.

The focus of this course is on the student-centered approach to the topics during the seminary part. In the 3-day workshop, the participants apply their knowledge to a specific problem on earthquake responses in multi-storey buildings. Most parts of the so-called simulation pipeline are, thus, tackled in a hands-on approach:

- Theory
- Modeling
- Numerics & Algorithms
- Implementation & Data Structures
- Visualization
- Verification & Validation

The major innovative element of this course is the workshop: The task is to realize a small software product in Matlab. The participants will experience (parts of) the challenges and requirements of interdisciplinary projects in academia and industry.

The seminary talks are chosen from the following topics.

Dynamical Systems & Randomly Perturbed Differential Equations

- Classical theory of ordinary differential equations (ODE) and corresponding numerics
- Dynamical systems and stability of equilibria
- Solution concepts of random differential equations
- Numerics of random differential equations
- Stochastic stability

Algorithms of Scientific Computing

- Reduction or partial differential equations (PDE) to systems of ODE using spatial discretisation
- Basic numerical methods for (deterministic) ODE
- Fourier transform
- Discrete Sine transform
- Space-filling curves

Application: Modeling seismic activities for buildings

- Modeling the dynamics of elastic bodies
- Relevant aspects of Software Engineering
- Visualization techniques

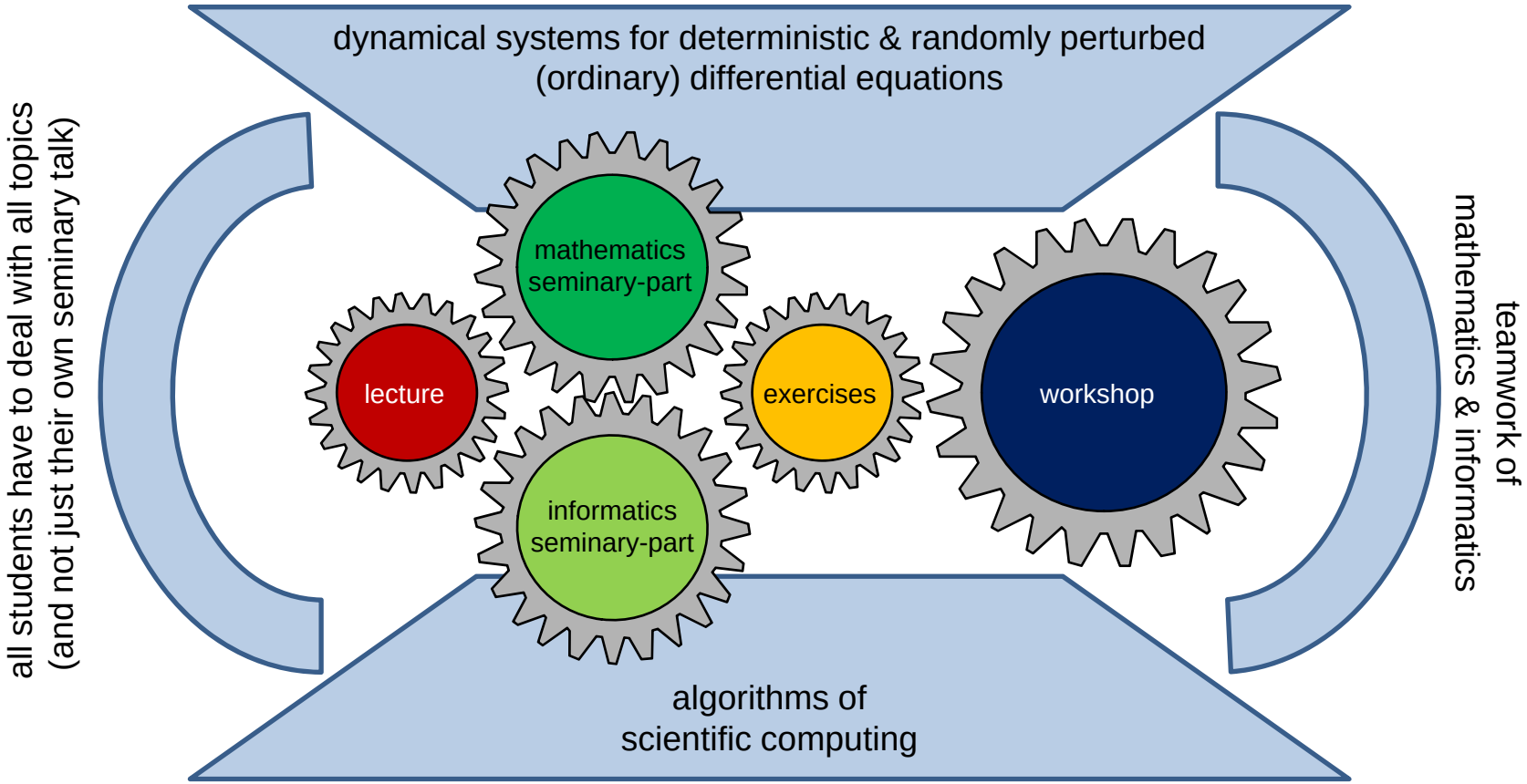
Dynamical Systems & Scientific Computing is part of the TUM informatics curriculum, module number ???
Semesterwochenstunden: 6 SWS
ECTS Credits: 9 Credits

The course integrates interdisciplinary, student-centered and application oriented learning …

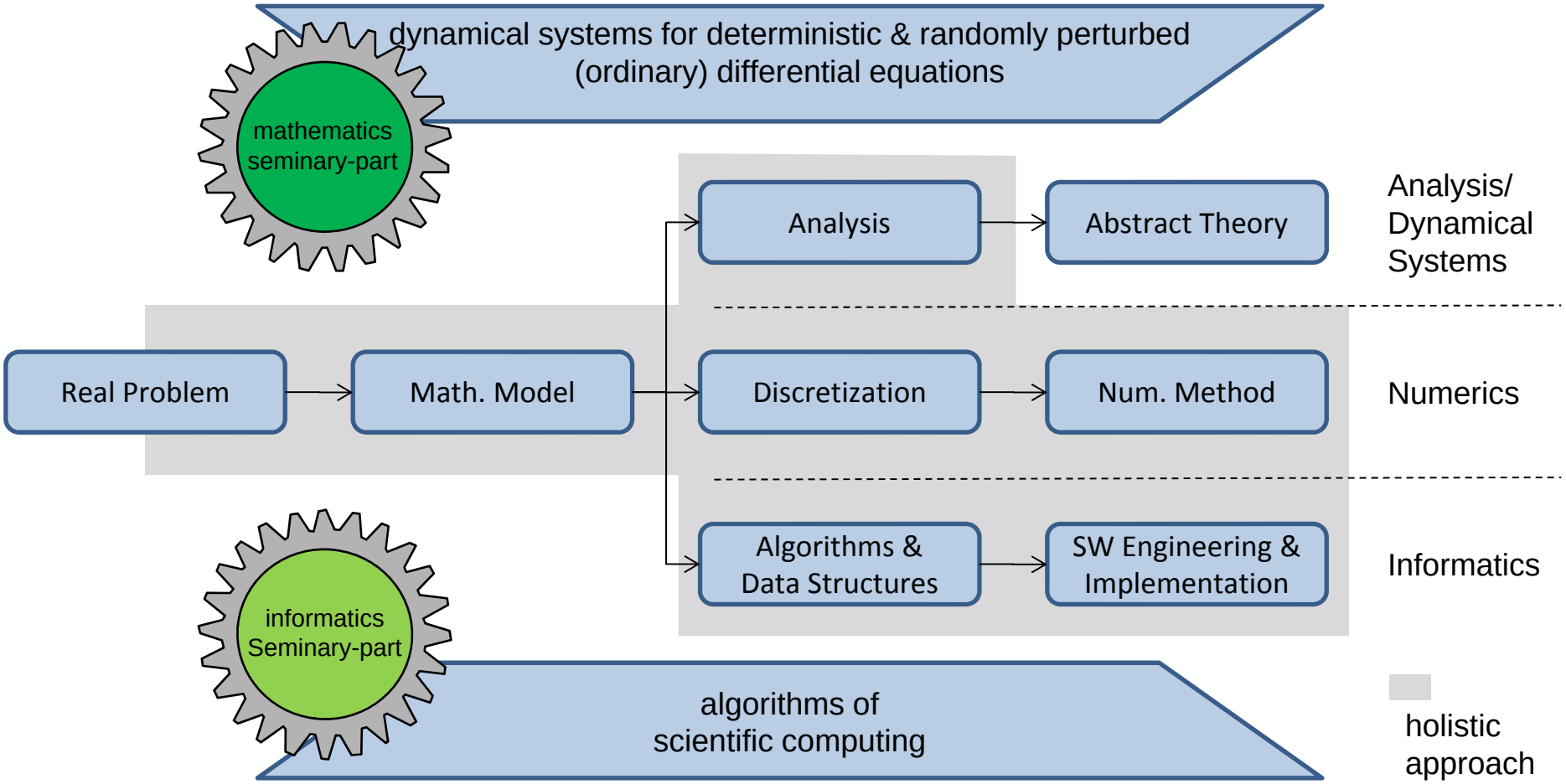
inter-disciplinary	• Topics from the intersection of mathematics and informatics • Course for master students of mathematics as well as informatics	**Challenges:** • self-contained and relevant topic • various levels of previous knowledge
student-centered	• Generation of a stimulating learning environment • Learning as a continuous process and not just for the exam	**Challenges:** • leave traditional lecture routine • motivating students to personally contribute
application oriented	• Exciting examples and problems • Teamwork • Realistic work assignments for the students	**Challenges:** • definition of a new course format • definition of “realistic” scenarios

… how do we want to realize this?

… and thus combines different (traditional) structural elements in an innovative way



Thematically, theory and numerics of random differential equations (RDEs) are studied in a holistic approach



RDEs play a key role in modern ingenieering science

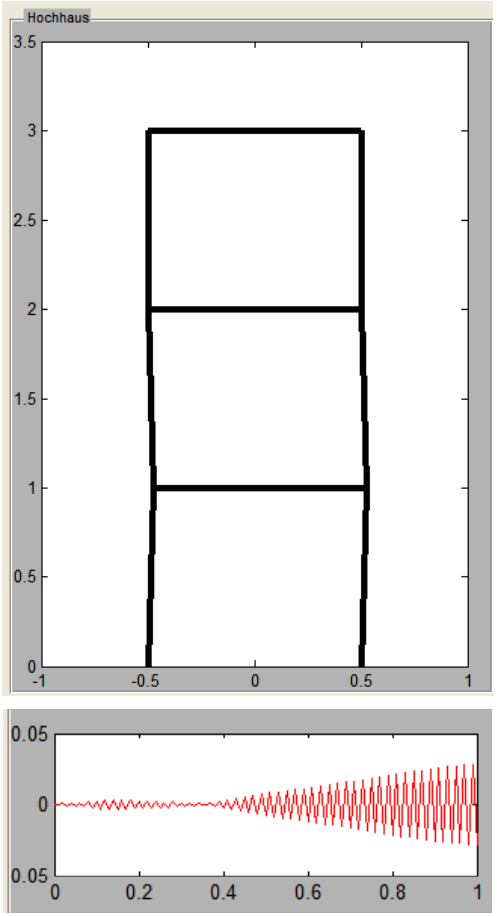
What is a random (ordinary) differential equation (RDE)?

$$\dot{x}(t) \;=\; f(t, x(t), \xi_t)$$

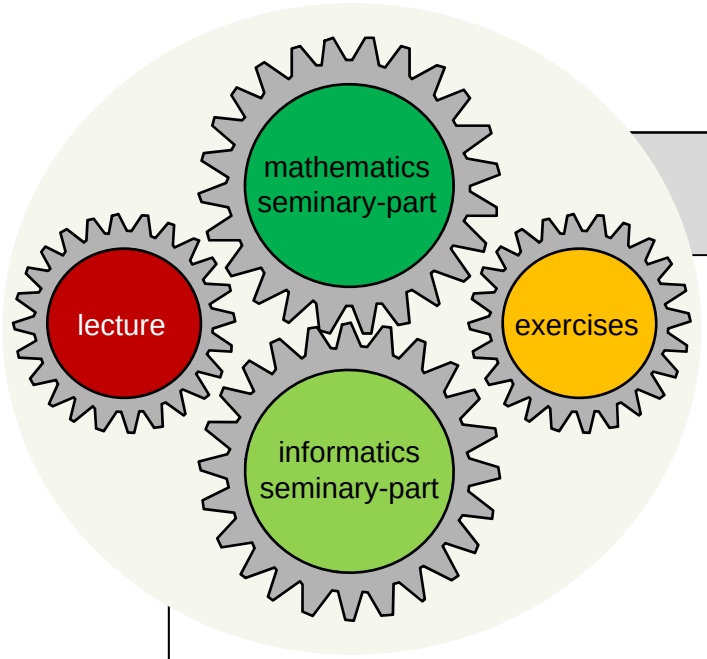
- ξ_t represents a continuous stochastic process
- The solution x(t) has continuous and differentiable paths
- The solution can path-wise be interpreted as an ODE

Where do such equations occur naturally?

- Everywhere where real noise processes (colored noise) with finite spectral energy occur, and where the artificial white noise is an approximation only
- At the description of the behavior of mechanic structures and their deformation (incl. hysteresis effects) subject to earthquakes, waves, wind, …



The combination of the seminary with lecture parts and, in particular, with exercises secures the learning progress sustainably



Quality is ensured by intensive preparation as well as additional exercises and deepening lectures

Situation: A well known disadvantages of seminaries and reading courses is that the participants – more or less – just prepare their own; during the rest of the course they are (due to the nature of a seminary) rather passive.

Solution: Seminaries have to be thrilling and all participants have to deal with all topics

- Small **lecture** part functions as additional deepening possibility and quality-insurance
- For each seminary talk there will be an elaborate **handout** and a **rehearsal talk**
- **Exercises**: teams consisting each of one student of mathematics and one student of informatics solve the application oriented problems

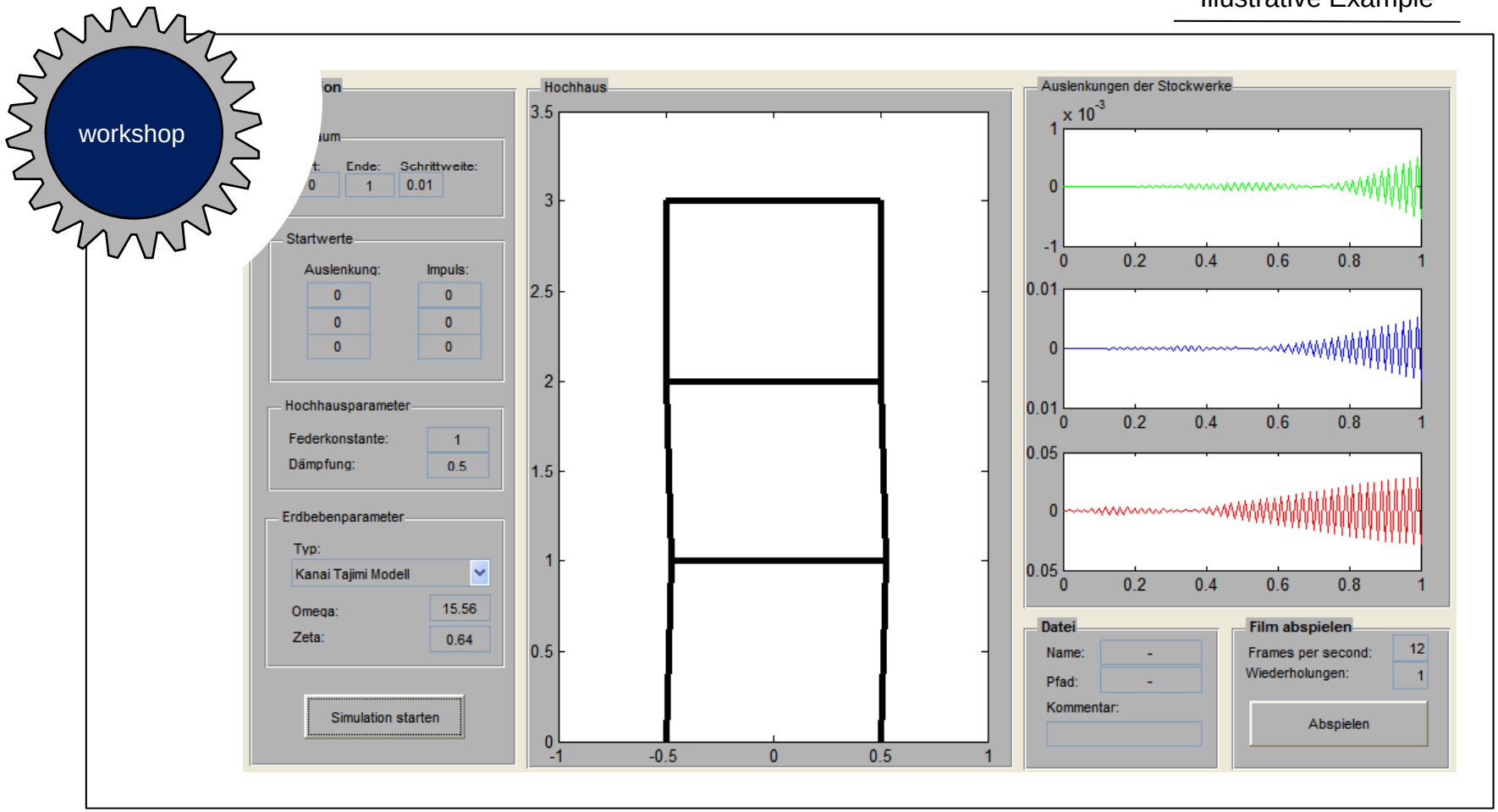
The workshop constitutes the highlight of the course and allows for learning the whole „simulation-pipeline“

mathematics & informatics as a team: “software-company”

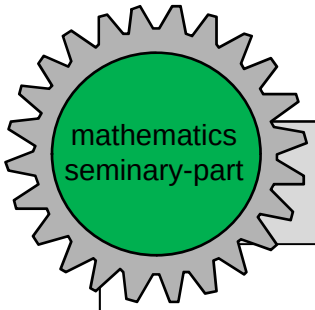
1. **Professional deepening:**
   - concrete practical context
   - experience the “big picture” of structures subject to ground motion excitations
2. **Intensification of the interdisciplinary cooperation:**
   entire team and sub-teams respectively: mingling of the participants from the two different fields of study
3. **Hands-on team-/ project experience:**
   practicing team-/ project situations in a save environment though under real conditions:
   - professionally challenging tasks & work packages not just the usual toy examples
   - concrete demands and specifications of a “customer”
   - time pressure and limited resources to execute the tasks

A possible result of the software procjet realized at the workshop would display the earthqualte response of a multi-story buliding

Illustrative Example

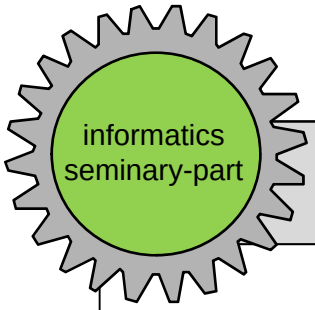


The seminary topics impart solid foundations and optimally support the realization of the workshop



focus on foundations of RDEs

- Path-Wise Solutions of RDEs
- Theory of ODEs
- Deterministic Dynamical Systems
- Stability of ODE Solutions
- Stability of Linear Systems and Stability of Matrices
- The Shadowing Lemma and Numerical Approximation of Solutions
- Properties of Solutions of RDEs
- Stability of Path-Wise Solutions
- Simulating Path-Wise Solutions



focus on efficient algorithms

- Material Laws
- Reduction of PDEs to RDEs
- Simulation of ODEs
- Visualization Principles
- Space Filling Curves
- Frequency Domain and Fourier Transform
- Propagation of Oscillations and the Response Spectrum of Structures
- Discrete Sine Transform
- Software Engineering

PDE: Partial Differential Equation
ODE: Ordinary Differential Equation
RDE: (Ordinary) Random Differential Equation

Detailed planning of the seminary topics (1/ 6)

Decomposition of Multi-Story Building Excitation Problems

Material Laws (TN, 1 Talk)
- Basics of Continuum Mechanics
- Derivation of the Underlying Elasticity Differential Equations
- Choice of Parameters
- Steady State vs. Time Dependency

Reduction of Partial Differential Equations to Random Differential Equations (TN, 1 Talk)
- Modeling Principles
- The Problem of Multi-Story Building Excitations
- Elliptic and Parabolic Partial Differential Equations
- Finite Difference Approximations

Path-Wise Solutions of Random Differential Equations (FR, 1 Talk)
- Modeling External Excitations as Bounded Stochastic Processes
- Random Differential Equations and Their Solution Concepts
- Existence and Uniqueness of Path-Wise Solutions
- Simulation of Solution Paths: The Explicit Euler-Schema
- The Mathematical Oscillator and Limitations of Non-Symplectic Schemas

TN: seminary talk supervised by Tobias Neckel
FR: seminary talk supervised by Florian Rupp

Detailed planning of the seminary topics (2/ 6)

The Path-wise Deterministic Setting (1/ 2)

Theory of Ordinary Differential Equations (FR, 1 Talk)
- Existence and Uniqueness of Solutions
- Dependence on Parameters and Initial Conditions
- Analytic Mechanics and Hamiltonian Systems
- The Mathematical Pendulum

Simulation of Ordinary Differential Equations (TN, 1 Talk)
- The Implicit Euler-Schema
- Explicit Runge-Kutta-Schemas
- Implicit Runge-Kutta-Schemas
- Symplectic Schemas and Numerics for the Mathematical Pendulum

Visualization Principles (TN, 1 Talk)
- Visualization Techniques (higher dimensional data)
- Postprocessing Tools (vtk/ paraview, visit, tecplot)
- GUI Usage/ Development (Qt, guide)

TN: seminary talk supervised by Tobias Neckel
FR: seminary talk supervised by Florian Rupp

Detailed planning of the seminary topics (3/ 6)

The Path-wise Deterministic Setting (2/ 2)

Deterministic Dynamical Systems and Stability of Solutions (FR, 1 Talk)
- Basic Definitions
- Attraction and Stability
- Lyapunov Functions and Non-Linear Stability
- Existence of Lyapunov Functions and Zubov's Equation

Stability of Linear Systems and Stability of Matrices (FR, 1 Talk)
- The Principle of Linearized Stability
- Lyapunov Exponents and the Stability of Linear Systems
- Classification of Linear Systems
- Sturm Sequences and the Routh-Hurwitz-Criterion
- Lozinskii Measure and Asymptotic Stability

The Shadowing Lemma and Numerical Approximation of Solutions (FR, 1 Talk)
- Basic Definitions (Pseudo-Orbits & Shadowing)
- The Discrete and Continuous Shadowing Lemma
- Proof of the Shadowing Lemma (in the Linear Case)
- Numerical Shadowing

TN: seminary talk supervised by Tobias Neckel
FR: seminary talk supervised by Florian Rupp

Detailed planning of the seminary topics (4/ 6)

Path-Wise Solutions of Random Differential Equations and Their Simulation

Properties of Solutions of Random Differential Equations (FR, 1 Talk)
- Dependence on Parameters and Initial Conditions
- Recurrence and Transience of Solutions
- Explosion of Solutions
- Explicit Solutions of Linear Random Differential Equations

Simulating Path-Wise Solutions (FR, 1 Talk)
- The Monte Carlo Algorithm
- Quasi Monte-Carlo Methods
- Stochastic Euler and Stochastic Runge-Kutta Schemas

Stability of Path-Wise Solutions (FR, 1-2 Talks)
- Fundamental Stability Definitions for Random Differential Equations
- Connections Between These Stability Definitions
- Stability of Linear Random Differential Equations
- Sufficient Conditions for Path-Wise Stability by Laypunov's Second Method
- Stability Subject to Permanently Acting Excitations

TN: seminary talk supervised by Tobias Neckel
FR: seminary talk supervised by Florian Rupp

Detailed planning of the seminary topics (5/ 6)

Efficient Data Structures and the Propagation of Random Excitations (1/ 2)

Frequency Domain and Fourier Transform (TN, 1 Talk)
- The Frequency Domain
- Linear Filters and the Response Spectrum
- Continuous Fourier Transform
- Discrete Fourier Transform
- The Fast Fourier Transform (FFT) Algorithm

The Discrete Sine Transform (TN, 1 Talk)
- Solving Partial Differential Equations by FFT
- The Sine Transform
- Response of a Multi Degree of Freedom Frame Structure to Random Excitations

Propagation of Oscillations and the Response Spectrum of Structures (FR, 1-2 Talks)
- Response of Single Degree of Freedom System to Transient Loads
- Random Excitations and Response
- Response of Elastic Structures to Random Excitations
- Response of a Frame Structure to Random Excitations

TN: seminary talk supervised by Tobias Neckel
FR: seminary talk supervised by Florian Rupp

Detailed planning of the seminary topics (6/ 6)

Efficient Data Structures and the Propagation of Random Excitations (2/ 2)

Space Filling Curves (TN, 1 Talk)
- Motivation: (Cartesian) Grids and Trees
- Definition of Space Filling Curves
- The Hilbert Curve (2D, 3D, Grammar, Indexing)

Software Engineering (TN, 1 Talk)
- Versioning Systems (CVS, svn)
- Methodology (survey of models, best practices)
- UML and its application to Matlab
- Automated Tests and Continuous Integration

TN: seminary talk supervised by Tobias Neckel
FR: seminary talk supervised by Florian Rupp

The course shedule is designed to avoid collissions with other exams

Schedule for the Summer Term 2012

April			May			June			July		
1 Su		CW 14	1 Tu			1 Fr	Consultation-Hour		1 Su		CW 27
2 Mo			2 We	ODE Numerics		2 Sa			2 Mo	Evaluation & Feedback	
3 Tu			3 Th			3 So		CW 23	3 Di		
4 We			4 Fr	Sprechstunde		4 Mo	Discrete Sine Transfrom		4 Mi	SC Questions & Answers	
5 Th			5 Sa			5 Di			5 Do		
6 Fr			6 Su		CW 19	6 Mi	Space Filling Curves		6 Fr	Exam Q & A	
7 Sa			7 Mo	DS Basics		7 Do			7 Sa		
8 Su		CW 15	8 Tu			8 Fr	Exercises (Sheet 4)		8 Su		CW 28
9 Mo			9 We	Basics of Visulaization		9 Sa			9 Mo		
10 Tu			10 Th			10 So		CW 24	10 Di	autonomous	
11 We			11 Fr	Exercises (Sheet 2)		11 Mo	RDE Stability 1		11 Mi	recap of the lecture	
12 Th			12 Sa			12 Di			12 Do	within the teams	
13 Fr			13 Su		CW 20	13 Mi	RDE Stability 2		13 Fr		
14 Sa			14 Mo	Stability Theory		14 Do			14 Sa		
15 Su		CW 16	15 Tu			15 Fr	Workshop-Spezifications		15 Su		CW 29
16 Mo	Introduction		16 We	The Shadowing Lemma		16 Sa			16 Mo	Consolidation: Stoch. Proc.	
17 Tu			17 Th			17 So		CW 25	17 Di		
18 We	Material Laws		18 Fr	Consultation-Hour		18 Mo	Software Engineering		18 Mi	DS Questions & Answers	
19 Th			19 Sa			19 Di			19 Do		
20 Fr	Presentation Skills		20 Su		CW 21	20 Mi	Workshop Preparation		20 Fr	Exam Q & A	
21 Sa			21 Mo	RDE Theory		21 Do			21 Sa		
22 Su		CW 17	22 Tu			22 Fr			22 Su		CW 30
23 Mo	PDE Forumlation		23 We	RDE Numerics		23 Sa	Workshop		23 Mo	Exam	
24 Tu			24 Th			24 So		CW 29	24 Tu		
25 We	RDE Formulation		25 Fr	Exercises (Sheet 3)		25 Mo	Workshop Wrap-up		25 We		
26 Th			26 Sa			26 Di			26 Th		
27 Fr	Exercises (Sheet 1)		27 Su		CW 22	27 Mi	Summary		27 Fr		
28 Sa			28 Mo			28 Do			28 Sa		
29 Su		CW 18	29 Tu			29 Fr	Exercises (Sheet 5)		29 Su		CW 31
30 Mo	ODE Theory		30 We	Fast Fourier Transform		30 Sa			30 Mo		
			31 Th						31 Tu		

(Classical) Lecture

Seminary (DS Focus)
Seminary (SC Focus)

Exercises/ Consultation-Hours

Workshop

ODE	Ordinary Differential Equations
PDE	Partial Differential Equations
RDE	Random Differential Equations
DS	Dynamical Systems
SC	Scientific Computing

Scheduling the course

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00					
10:00					
11:00					
12:00					
13:00					
14:00					
15:00					
16:00					
17:00					
18:00					

preferred seminary slot
alternative seminary slot
preferred exercise slot
alternative exercise slot

Grading-Schema

The total grade of the course will be calculated as a weighted sum of

- individual achievements (talk, handout and exam), and
- team achievements during the workshop

$$Grade = \left\lfloor \frac{5 \cdot Exam}{15} + \frac{4 \cdot Talk + 2 \cdot Handout + 4 \cdot Workshop}{15} \right\rfloor$$

Decimal points in the final grade will rounded to the next lower decimal .0, .3 or .7.

To ensure that your talk and your handout will be a success visit us regularly during your preparation. Moreover, an informal rehearsal talk is mandatory.

The best preparation for the workshop and the exam are the exercises that you will get during the course.

What we expect from you

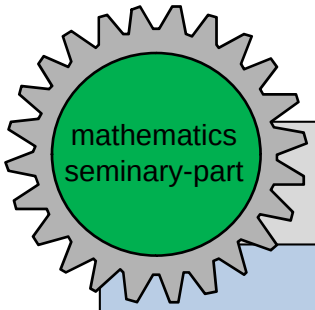
Task	Deadline	Grading
Preparation of the talk		
• handout	at least 2 weeks prior to your talk	2/15
• rehearsal talk	at least 1 week prior to your talk	
• talk	t.b.d.	4/15
Proactive participation	ongoing	
Exercises	ongoing	
Workshop	see schedule	4/15
Exam	see schedule	5/15

Next steps (during the break)

Task	Responsible	Due to
Choose your seminary topic	you	a.s.a.p.
Contact your TN or FR to …		
• … define the talks' scope	you & TN, FR	a.s.a.p.
• … get relevant materials	you & TN, FR	1st of March
Prepare your …		
• … handout	you	t.b.d.
• … talk	you	t.b.d.

Recent news will be displayed at the website of the course:
www5.in.tum.de/wiki/index.php/Dynamical_Systems_%26_Scientific_Computing_-_Summer12

… and your chance to take action and opt for a seminary topic



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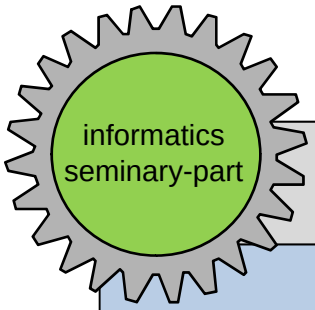

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- Software Engineering

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thank you very much for your attention