Dynamical Systems & Scientific Computing: Homework Assignments

3.1 [☼] **(Application of the Routh-Hurwitz Method)**
Apply the Routh-Hurwitz method to determine the location of all roots of the following polynomials:

a) $p(x) = 3x + 5$,
b) $p(x) = -2x^2 - 5x - 100$,
c) $p(x) = 523x^2 - 57x + 189$,
d) $p(x) = (x^2 + x - 1)(x^2 + x + 1)$, and
e) $p(x) = x^3 + 5x^2 + 10x - 3$.

3.2 [☼] **(Implementation of the Routh-Hurwitz Algorithm)**
Implement the Routh-Hurwitz method for determining the location of the roots of a polynomial of degree 4 within the complex plane in Matlab. Use the Matlab root finding methods to plot the roots in the complex plane.

a) Test your code against the analytic results derived in exercise 3.1.
b) Apply your code to determine the stability of the zero solution of the linear oscillator differential equation
$$\ddot{x}(t) + 2cd\dot{x}(t) + d^2x(t) \;=\; 0\,, \qquad x \in \mathcal{C}^2(\mathbb{R},\mathbb{R})\,,$$
with constants $c = 0.64$ and $d = 15.56$, cf. exercise 2.4.
c) Finally, consider the parameters c and d from part b) to be arbitrary, i.e., $(c,d) \in \mathbb{R}^2$. Define a function depending on c and d that is positive if the origin is asymptotically stable in exercise b), zero if the origin is stable and negative if the origin is unstable. Plot the graph of this function against the c-d-plane.

3.3 [☼] **(Application of the Lozinskii-Measure)**
In an article by Rupp and Scheurle ([3]) a reduced model for fish-jellyfish interactions is proposed, where fish being assumed to represent the dominant predatory species feeding on jellyfish. This model is given by the following set of coupled non-linear ordinary differential equations
$$\dot{x} \;=\; \left(c + \frac{y}{1+y}\right)x - x^2\,, \quad \text{and} \quad \dot{y} \;=\; \beta\left(1 - d\frac{x}{1+y}\right)y\,,$$
where $c \in \mathbb{R}$ and $\beta, d > 0$ are parameters, and x denotes the fish population whereas y stands for the jellyfish population, respectively.

a) Show that the origin is an equilibrium and give the linearization of the equations at this point.
b) Apply the method of Lozinskii-Measures to determine (linearized) stability of the origin.

3.4 [☼] **(RDE Properties)**

In order to describe in some qualitative manner the nature of an earthquake disturbance, Bogdanoff et al. [1], [2], considered the following model for ground accelerations

$$y(t) = \sum_{j=1}^{n} t a_j \exp(-\alpha_j t)\cos(\omega_j t + \Theta_j)\,, \quad \text{for } t \geq 0\,, \qquad \text{and} \qquad y(t) = 0 \quad \text{for } t < 0\,, \tag{1}$$

where a_j, α_j and ω_j are given real positive numbers and the parameters Θ_j are independent random variables uniformly distributed over an interval of length 2π.

Let us assume a one-story building that is at rest at $t = 0$ and let $X(t)$, $t \geq 0$, denote the relative horizontal displacement of its roof with respect to the ground. Then, based upon an idealized linear model, the relative displacement $X(t)$ subject to ground accelerations is governed by

$$\ddot{x}(t) + 2\zeta\omega_0\dot{x}(t) + \omega_0^2 x(t) = -y(t)\,, \quad \text{for } t \geq 0\,. \tag{2}$$

a) Show that (2) has a path-wise solution.

b) Compute this solution analytically.

3.5 [☼] **(RDE Simulation)**

For equation (2) let the parameters be given according to [2], p.267, as $\omega_0 = 20$ [rad/ sec] and $\zeta = 0.05$.

a) Let $y(t)$ be the stochastic process defined by (1) the coefficients of which are given according to [2], p.267, as $\omega_1 = 6$ [rad/ sec], $\omega_2 = 8$ [rad/ sec], $\omega_3 = 10$ [rad/ sec], $\omega_4 = 11.15$ [rad/ sec], $\omega_5 = 20,75$ [rad/ sec], $\omega_6 = \ldots, \omega_{11} = 0$ [rad/ sec], $\omega_{12} = 21.50$ [rad/ sec], $\omega_{13} = 23.25$ [rad/ sec], $\omega_{14} = 25$ [rad/ sec], $\omega_{15} = 27$ [rad/ sec], $\omega_{16} = 29$ [rad/ sec], $\omega_{17} = 30.5$ [rad/ sec], $\omega_{18} = 32$ [rad/ sec], $\omega_{19} = 34$ [rad/ sec], $\omega_{20} = 36$ [rad/ sec], as well as $\alpha_1 = \cdots = \alpha_{20} = 1/3$, $a_1 = \cdots = a_{20} = 0.5$.

Plot $y(t)$ against the time $t \in [0, 10]$.

b) Apply the usual (deterministic) Euler scheme to solve (2) for the given parameter values.

c) Apply the usual (deterministic) Heun scheme to solve (2) for the given parameter values.

d) Compare the solution from b) and c) with the analytical solution obtained in 3.4 b).

Classification: ☼ easy, ◷ easy with longer calculations, ☆ a little bit difficult, ◫ challenging.

References

[1] J.L. Bogdanoff, J.E. Goldberg and M.C. Bernard (1961): *Response of a Simple Structure to a Random Earthquake-Type Disturbance*, Bull. Seism. Soc. Amer., 51, pp. 293-310.

[2] J.E. Goldberg, J.L. Bogdanoff, and D.R. Sharpe (1964): *The Response of Simple Nonlinear Systems to Random Disturbance of the Earthquake Type*, Bull. Seism. Soc. Amer., 54, pp. 263-274.

[3] F. Rupp and J. Scheurle (2012): *On the Jellyfish Joyride: Mathematical Analysis of Catastrophes in Maritime Ecosystems*, to appear at the proceedings of the 9th AIMS Conference on Dynamical Systems.