

# Dynamical Systems & Scientific Computing



Ernst Otto Fischer-Lehrpreis der TUM

**Ernst Otto Fischer-Lehrpreis 2011  
der TU München**

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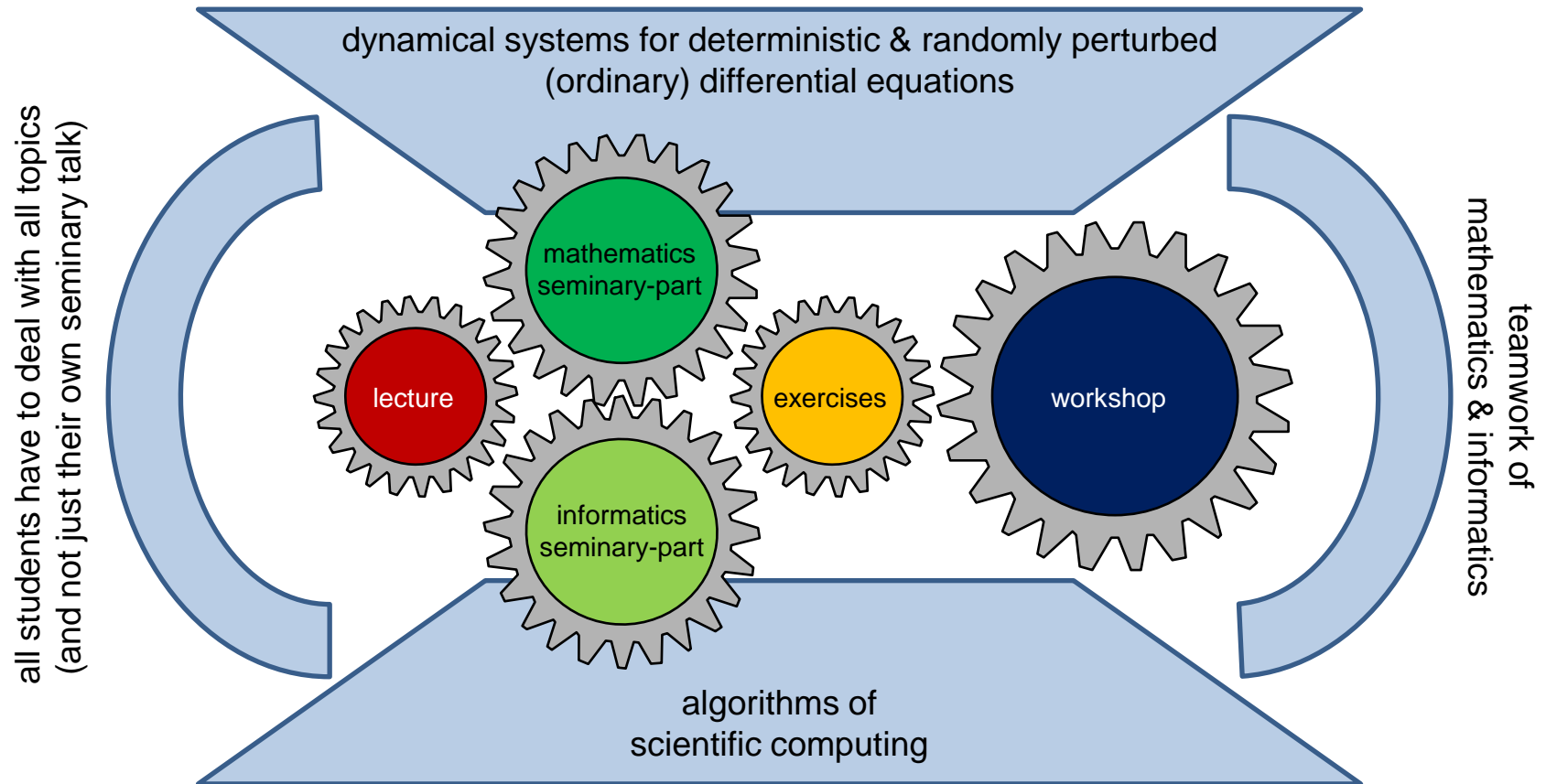


## The course integrates interdisciplinary, student-centered and application oriented learning ...

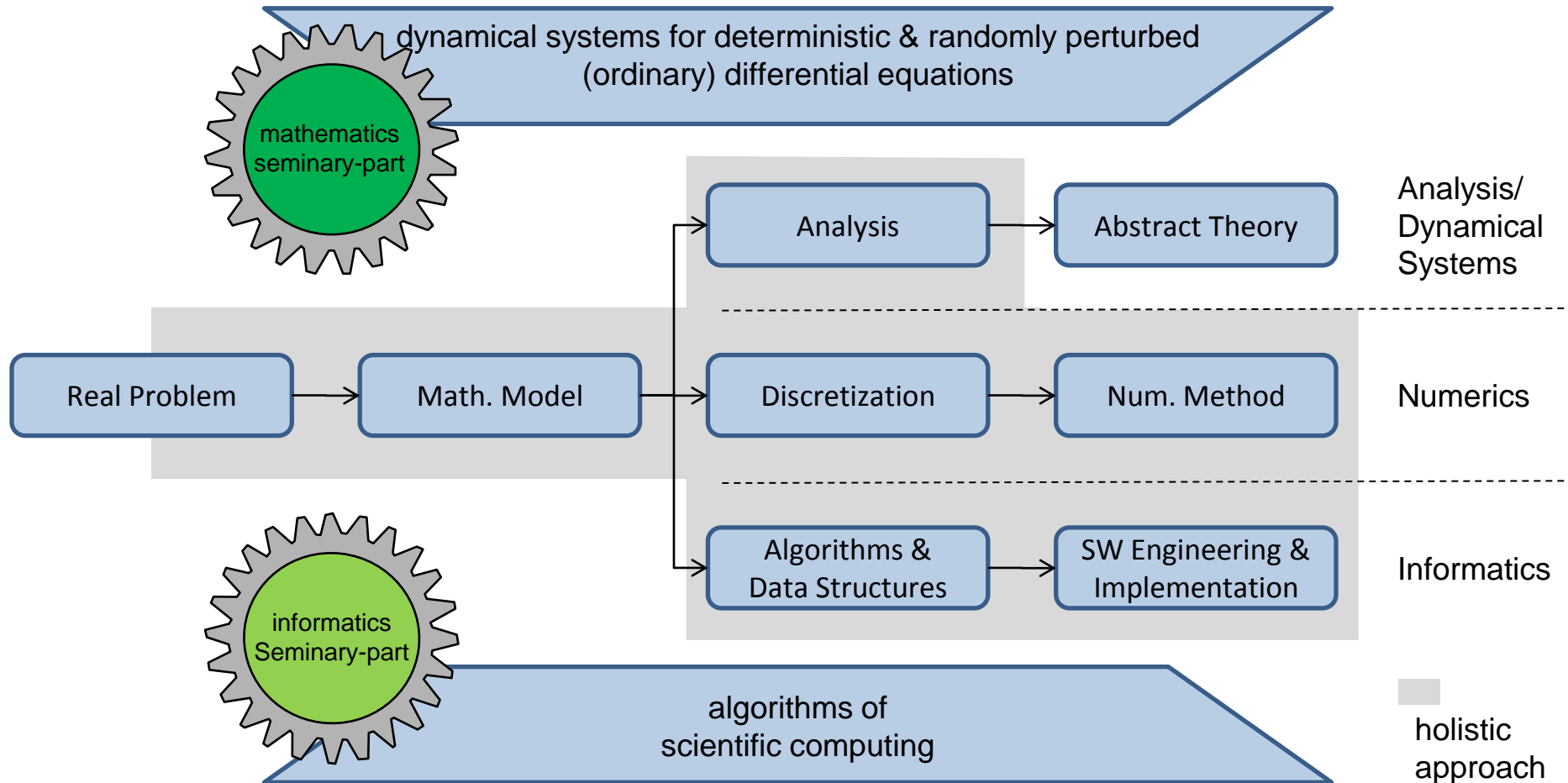
<b>inter- disciplinary</b>	<ul style="list-style-type: none"><li>• Topics from the intersection of mathematics and informatics</li><li>• Course for master students of mathematics as well as informatics</li></ul>	<b>Challenges:</b> <ul style="list-style-type: none"><li>• self-contained and relevant topic</li><li>• various levels of previous knowledge</li></ul>
<b>student- centered</b>	<ul style="list-style-type: none"><li>• Generation of a stimulating learning environment</li><li>• Learning as a continuous process and not just for the exam</li></ul>	<b>Challenges:</b> <ul style="list-style-type: none"><li>• leave traditional lecture routine</li><li>• motivating students to personally contribute</li></ul>
<b>application oriented</b>	<ul style="list-style-type: none"><li>• Exciting examples and problems</li><li>• Teamwork</li><li>• Realistic work assignments for the students</li></ul>	<b>Challenges:</b> <ul style="list-style-type: none"><li>• definition of a new course format</li><li>• definition of “realistic” scenarios</li></ul>

... how do we want to realize this?

... and thus combines different (traditional) structural elements in an innovative way



# Thematically, theory and numerics of random differential equations (RDEs) are studied in a holistic approach



# RDEs play a key role in modern ingenieering science

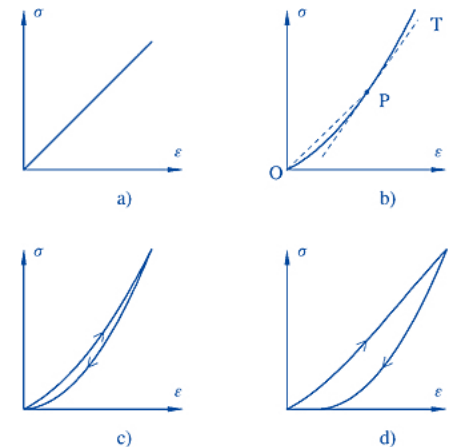
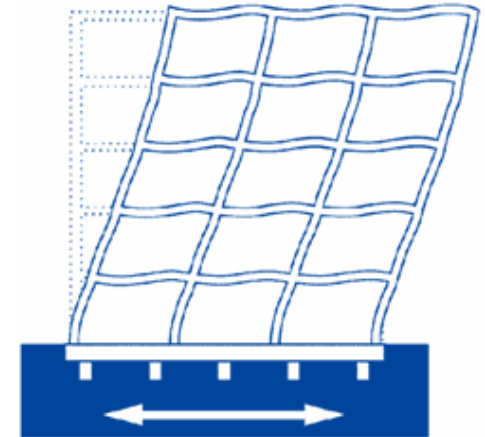
## What is a random (ordinary) differential equation (RDE)?

$$\dot{x}(t) = f(t, x(t), \xi_t)$$

- $\xi_t$  represents a continuous stochastic process
- The solution  $x(t)$  has continuous and differentiable paths
- The solution can path-wise be interpreted as an ODE

## Where do such equations occur naturally?

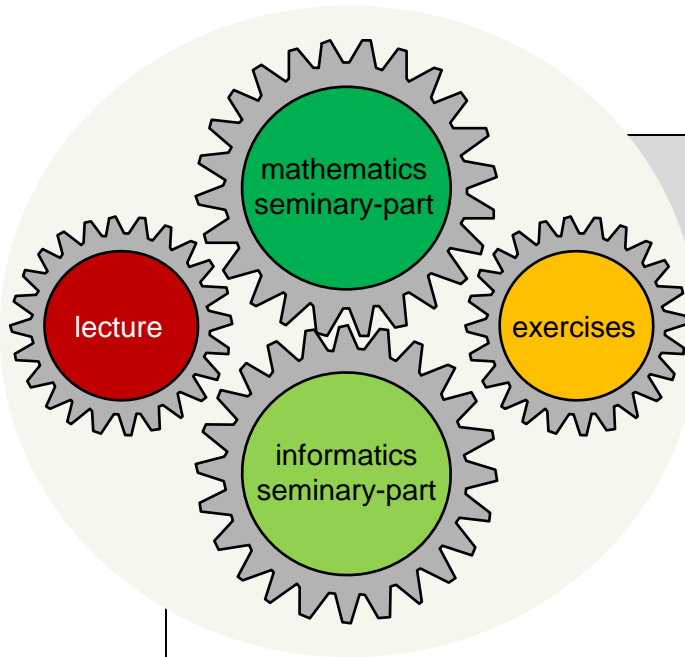
- Everywhere where real noise processes (colored noise) with finite spectral energy occur, and where the artificial white noise is an approximation only
- At the description of the behavior of mechanic structures and their deformation (incl. hysteresis effects) subject to earthquakes, waves, wind, ...



Impacts of seismic excitations on mechanic structures/ multi-story buildings (upper part) and sketch of relevant stress-strain relationships within these structures for:

- linear elastic materials,
- perfect elastic materials,
- elastic materials with hysteresis, and
- materials under permanent acting deforming forces.

The combination of the seminary with lecture parts and, in particular, with exercises secures the learning progress sustainably



Quality is ensured by intensive preparation as well as additional exercises and deepening lectures

**Situation:** A well known disadvantages of seminars and reading courses is that the participants – more or less – just prepare their own; during the rest of the course they are (due to the nature of a seminary) rather passive.

**Solution:** Seminars have to be thrilling and all participants have to deal with all topics

- Small **lecture** part functions as additional deepening possibility and quality-insurance
- For each seminary talk there will be an elaborate **handout** and a **rehearsal talk**
- **Exercises:** teams consisting each of one student of mathematics and one student of informatics solve the application oriented problems

The workshop constitutes the highlight of the course and allows for learning the whole „simulation-pipeline“



mathematics & informatics as a team: “software-company”

**1. Professional deepening:**

- concrete practical context
- experience the “big picture” of structures subject to ground motion excitations

**2. Intensification of the interdisciplinary cooperation:**

entire team and sub-teams respectively: mingling of the participants from the two different fields of study

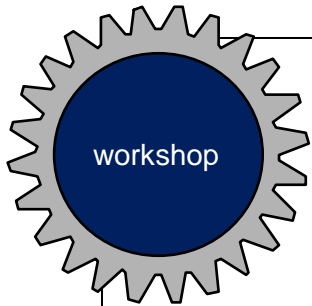
**3. Hands-on team-/ project experience:**

practicing team-/ project situations in a safe environment though under real conditions:

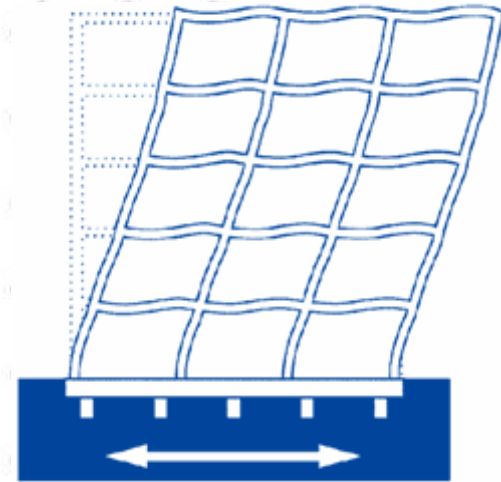
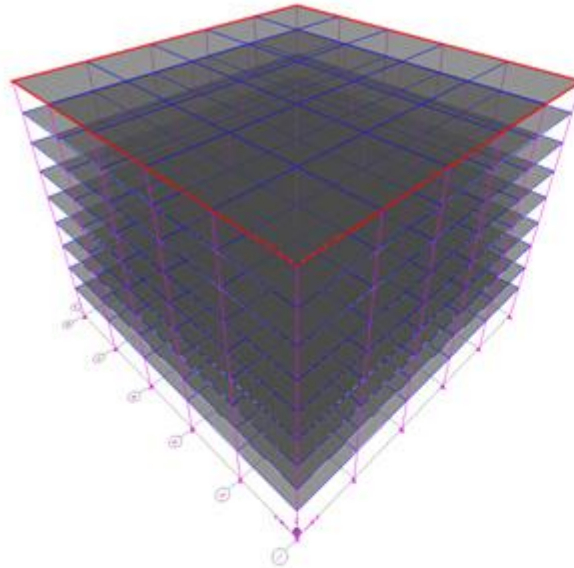
- professionally challenging tasks & work packages not just the usual toy examples
- concrete demands and specifications of a “customer”
- time pressure and limited resources to execute the tasks

# A possible result of the software project realized at the workshop would display the earthquake response of a multi-story building

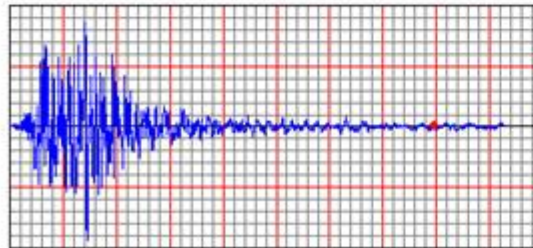
Illustrative Example



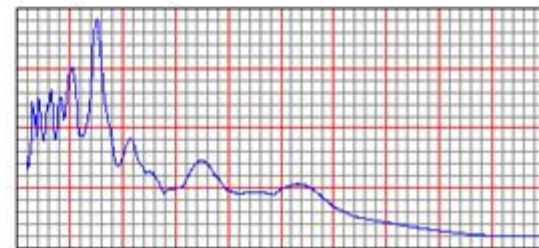
<http://www.stanford.edu/~kranthi/educationpics/dynamic.jpg>



Ground Motion:



Response Spectrum:





As a first step towards this goal, one may think of a GUI-based simulation for a building subject to random ground motion excitations ...

**Simulation**

Zeitraum  
Start: 0 Ende: 1 Schrittweite: 0.01

Startwerte  
Auslenkung: 0 Impuls: 0

Hochhausparameter  
Federkonstante: 1  
Dämpfung: 0.5

Erdbebenparameter  
Typ: Kanai Tajimi Modell  
Omega: 15.56  
Zeta: 0.64

Simulation starten

**Hochhaus**

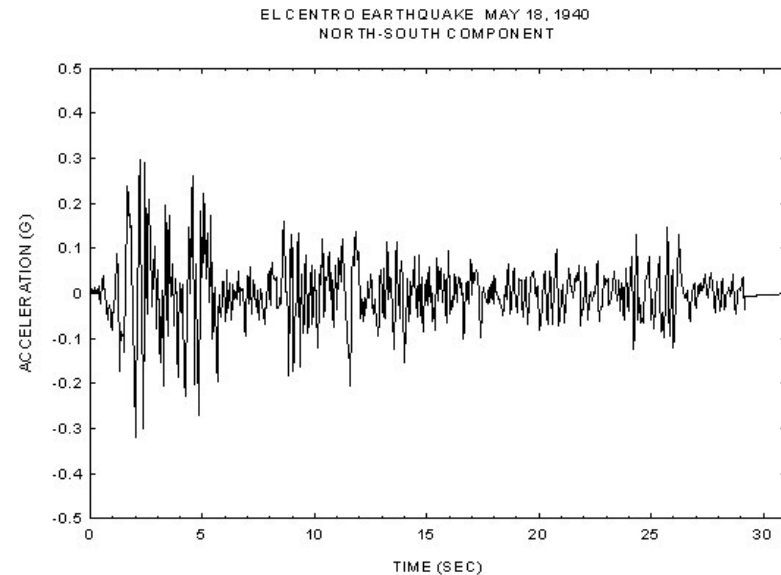
**Auslenkungen der Stockwerke**

$\times 10^{-3}$

Datei  
Name: -  
Pfad: -  
Kommentar:

Film abspielen  
Frames per second: 12  
Wiederholungen: 1  
Abspielen

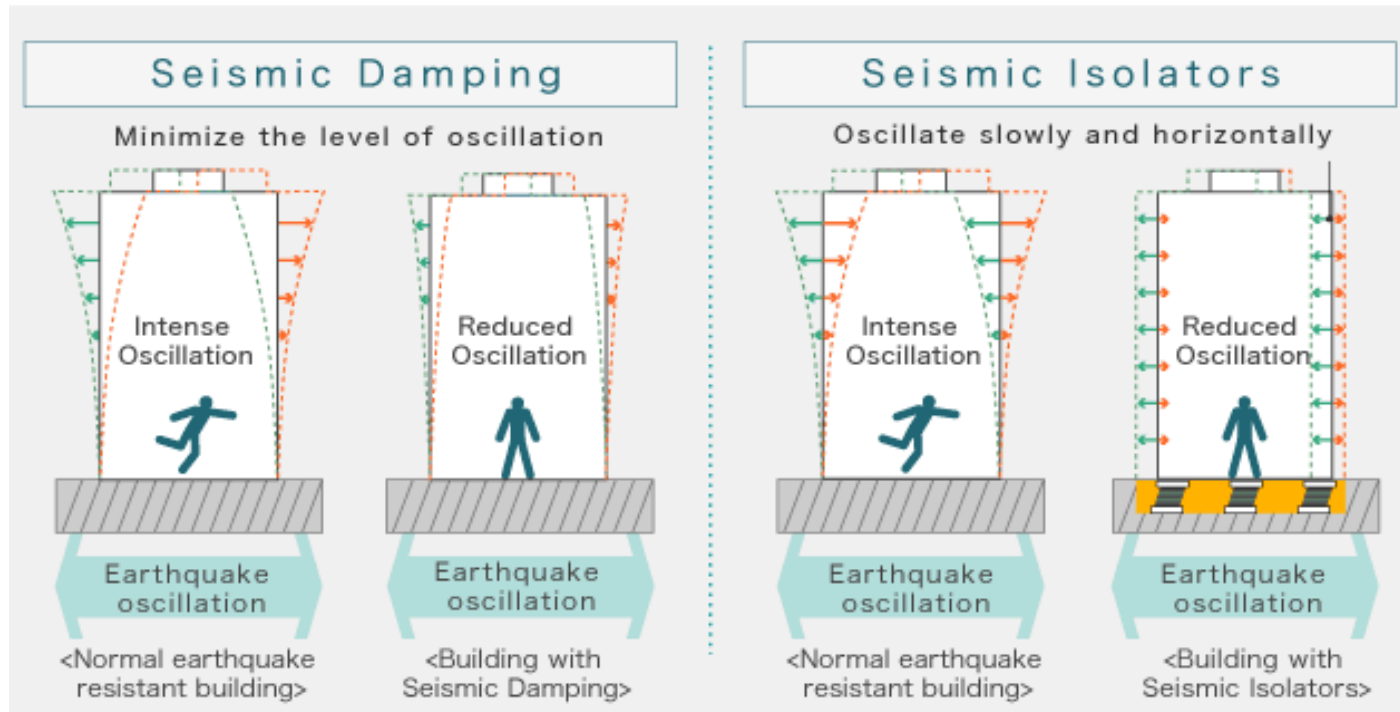
... that is extended such that it captures “real” effects like those observed at the El Centro Earthquake (1940) ...



### USGS Description of the El Centro (Imperial Valley) Earthquake

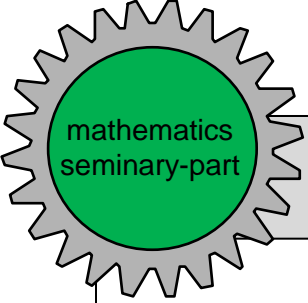
Nine people were killed by the May 1940 Imperial Valley earthquake. At Imperial, 80 percent of the buildings were damaged to some degree. In the business district of Brawley, all structures were damaged, and about 50 percent had to be condemned. The shock caused 40 miles of surface faulting on the Imperial Fault, part of the San Andreas system in southern California. It was the first strong test of public schools designed to be earthquake-resistant after the 1933 Long Beach quake. Fifteen such public schools in the area had no apparent damage. Total damage has been estimated at about \$6 million. The magnitude was 7.1.

... or such that it takes modern safety guidelines into account to minimize the devastating effect of a high-magnitude earthquake



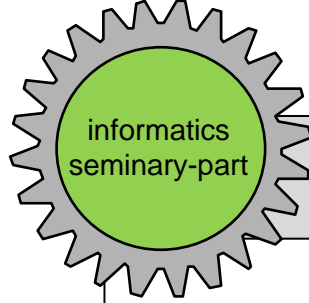
... pendulum attached at the top  
... water tank installed at the top  
... etc.

## The seminary topics impart solid foundations and optimally support the realization of the workshop



focus on  
foundations of RDEs

- Path-Wise Solutions of RDEs
- Theory of ODEs
- Deterministic Dynamical Systems
- Stability of ODE Solutions
- Stability of Linear Systems and Stability of Matrices
- The Shadowing Lemma and Numerical Approximation of Solutions
- Properties of Solutions of RDEs
- Stability of Path-Wise Solutions
- Simulating Path-Wise Solutions



focus on  
efficient algorithms

- Material Laws
- Reduction of PDEs to RDEs
- Simulation of ODEs
- Visualization Principles
- Space Filling Curves
- Frequency Domain and Fourier Transform
- Propagation of Oscillations and the Response Spectrum of Structures
- Discrete Sine Transform
- Software Engineering

PDE: Partial Differential Equation

ODE: Ordinary Differential Equation

RDE: (Ordinary) Random Differential Equation

# Detailed planning of the seminary topics (1/ 6)

## Decomposition of Multi-Story Building Excitation Problems

### **Material Laws (TN, 1 Talk)**

- Basics of Continuum Mechanics
- Derivation of the Underlying Elasticity Differential Equations
- Choice of Parameters
- Steady State vs. Time Dependency

### **Reduction of Partial Differential Equations to Random Differential Equations (TN, 1 Talk)**

- Modeling Principles
- The Problem of Multi-Story Building Excitations
- Elliptic and Parabolic Partial Differential Equations
- Finite Difference Approximations

### **Path-Wise Solutions of Random Differential Equations (FR, 1 Talk)**

- Modeling External Excitations as Bounded Stochastic Processes
- Random Differential Equations and Their Solution Concepts
- Existence and Uniqueness of Path-Wise Solutions
- Simulation of Solution Paths: The Explicit Euler-Schema
- The Mathematical Oscillator and Limitations of Non-Symplectic Schemas

# Detailed planning of the seminary topics (2/ 6)

## The Path-wise Deterministic Setting (1/ 2)

### **Theory of Ordinary Differential Equations (FR, 1 Talk)**

- Existence and Uniqueness of Solutions
- Dependence on Parameters and Initial Conditions
- Analytic Mechanics and Hamiltonian Systems
- The Mathematical Pendulum

### **Simulation of Ordinary Differential Equations (TN, 1 Talk)**

- The Implicit Euler-Schema
- Explicit Runge-Kutta-Schemas
- Implicit Runge-Kutta-Schemas
- Symplectic Schemas and Numerics for the Mathematical Pendulum
- Newmark-beta Method

### **Visualization Principles (TN, 1 Talk)**

- Visualization Techniques (higher dimensional data)
- Postprocessing Tools (vtk/ paraview, visit, tecplot)
- GUI Usage/ Development (Qt, guide)

# Detailed planning of the seminary topics (3/ 6)

## The Path-wise Deterministic Setting (2/ 2)

### **Deterministic Dynamical Systems and Stability of Solutions (FR, 1 Talk)**

- Basic Definitions
- Attraction and Stability
- Lyapunov Functions and Non-Linear Stability
- Existence of Lyapunov Functions and Zubov's Equation

### **Stability of Linear Systems and Stability of Matrices (FR, 1 Talk)**

- The Principle of Linearized Stability
- Lyapunov Exponents and the Stability of Linear Systems
- Classification of Linear Systems
- Sturm Sequences and the Routh-Hurwitz-Criterion
- Lozinskii Measure and Asymptotic Stability

### **The Shadowing Lemma and Numerical Approximation of Solutions (FR, 1 Talk)**

- Basic Definitions (Pseudo-Orbits & Shadowing)
- The Discrete and Continuous Shadowing Lemma
- Proof of the Shadowing Lemma (in the Linear Case)
- Numerical Shadowing

# Detailed planning of the seminary topics (4/ 6)

## Path-Wise Solutions of Random Differential Equations and Their Simulation

### **Properties of Solutions of Random Differential Equations (FR, 1 Talk)**

- Dependence on Parameters and Initial Conditions
- Recurrence and Transience of Solutions
- Explosion of Solutions
- Explicit Solutions of Linear Random Differential Equations

### **Simulating Path-Wise Solutions (FR, 1 Talk)**

- The Monte Carlo Algorithm
- Quasi Monte-Carlo Methods
- Stochastic Euler and Stochastic Runge-Kutta Schemas

### **Stability of Path-Wise Solutions (FR, 1-2 Talks)**

- Fundamental Stability Definitions for Random Differential Equations
- Connections Between These Stability Definitions
- Stability of Linear Random Differential Equations
- Sufficient Conditions for Path-Wise Stability by Laypunov's Second Method
- Stability Subject to Permanently Acting Excitations



# Detailed planning of the seminary topics (5/ 6)

## Efficient Data Structures and the Propagation of Random Excitations (1/ 2)

### **Frequency Domain and Fourier Transform (TN, 1 Talk)**

- The Frequency Domain
- Linear Filters and the Response Spectrum
- Continuous Fourier Transform
- Discrete Fourier Transform
- The Fast Fourier Transform (FFT) Algorithm

### **The Discrete Sine Transform (TN, 1 Talk)**

- Solving Partial Differential Equations by FFT
- The Sine Transform
- Response of a Multi Degree of Freedom Frame Structure to Random Excitations

### **Propagation of Oscillations and the Response Spectrum of Structures (FR, 1-2 Talks)**

- Response of Single Degree of Freedom System to Transient Loads
- Random Excitations and Response
- Response of Elastic Structures to Random Excitations
- Response of a Frame Structure to Random Excitations

# Detailed planning of the seminary topics (6/ 6)

## Efficient Data Structures and the Propagation of Random Excitations (2/ 2)

### **Space Filling Curves (TN, 1 Talk)**

- Motivation: (Cartesian) Grids and Trees
- Definition of Space Filling Curves
- The Hilbert Curve (2D, 3D, Grammar, Indexing)

### **Software Engineering (TN, 1 Talk)**

- Versioning Systems (CVS, svn)
- Methodology (survey of models, best practices)
- UML and its application to Matlab
- Automated Tests and Continuous Integration

## Grading-Schema

The total grade of the course will be calculated as a weighted sum of

- individual achievements (talk, handout and exam), and
- team achievements during the workshop

$$Grade = \left[ \frac{5 \cdot Exam}{15} + \frac{4 \cdot Talk + 2 \cdot Handout + 4 \cdot Workshop}{15} \right]$$

Decimal points in the final grade will rounded to the next lower decimal .0, .3 or .7.

To ensure that your talk and your handout will be a success visit us regularly during your preparation. Moreover, an informal rehearsal talk is mandatory.

The best preparation for the workshop and the exam are the exercises that you will get during the course.

## What we expect from you

Task	Deadline	Grading
Preparation of the talk		
• handout	at least 2 weeks prior to your talk	2/15
• rehearsal talk	at least 1 week prior to your talk	
• talk	t.b.d.	4/15
Proactive participation	ongoing	
Exercises	ongoing	
Workshop	see schedule	4/15
Exam	see schedule	5/15

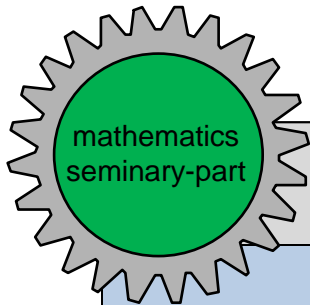
## Next steps (during the break)

Task	Responsible	Due to
Choose your seminary topic	you	a.s.a.p.
Contact your TN or FR to ...		
• ... define the talks' scope	you & TN, FR	a.s.a.p.
• ... get relevant materials	you & TN, FR	1 <sup>st</sup> of March
Prepare your ...		
• ... handout	you	t.b.d.
• ... talk	you	t.b.d.

Recent news will be displayed at the website of the course:

[www5.in.tum.de/wiki/index.php/Dynamical\\_Systems\\_%26\\_Scientific\\_Computing\\_-\\_Summer13](http://www5.in.tum.de/wiki/index.php/Dynamical_Systems_%26_Scientific_Computing_-_Summer13)

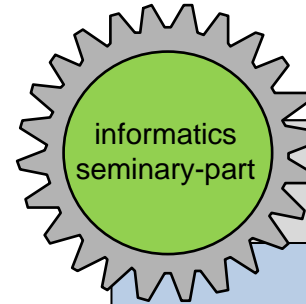
... and your chance to take action and opt for a seminary topic



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- Fast Fourier and Sine Transform
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# thank you very much for your attention

