# PSE Game Physics

## Session (7)
## Friction

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# Outline

## Friction types

- Static friction
- Dynamic friction
- Rolling friction
- Some coefficients

## Friction as impulses

- Change of velocity due to friction
- The modified velocity resolution algorithm

# Friction

- A Force generated when one object moves or tries to move in contact with another object.
- Two types of friction: static and dynamic friction.
- Results into resistance to the current motion.

# Static friction - Part 1

**Figure:** Example for static friction between an object and a surface

- A Force that stops an object from moving when it is stationary.
- Results into resistance to the current motion.

# Static friction - Part 2

- Depends on the material at the point of contact (the coefficient $\mu$) and the reaction force $r$ in the direction of the collision normal.

$$|f_{static}| \leq \mu_{static}|r|$$

- $\mu_{static}$ an empirical property described in tables for different pairs of materials.
- The reaction force $r$ is calculated :

$$r = -f.\vec{n},$$

  with $f$ the force acting on the object.
- The final equation for the static friction is:

$$f_{static} = \begin{cases} -f_{planar} \\ f_{planar} - \mu_{static}|r| \end{cases}$$

  the case with the smaller magnitude, with $f_{planar} = f + r$ the total force with the component in the direction of the contact normal removed.

# Dynamic friction - Part 1

**Figure:** Example for static friction between an object and a surface

- Has similar behaviour as the static friction.
- Described through different friction coefficient $\mu_{dynamic}$.
- For moving objects with contact between them.
- Less powerful.

# Dynamic friction - Part 2

- The equation for the dynamic friction depends on the velocity $\vec{v}$ of the moving object as follows:

$$f_{dynamic} = -\vec{v}_{planar}\mu_{dynamic}|r|$$

- What happens with the static and the dynamic friction, if we stop exerting a force on static object and on moving object?
- Most engines today mix the two types of friction.

# Rolling friction

- The friction observed when two objects roll along each other.
- Used for tire models for racing simulations.
- Ignored in this lab.

# Some coefficients

| Materials | Static friction | Dynamic friction |
|---|---|---|
| Wooden crate on concrete | 0.5 | 0.4 |
| Wooden crate on ice | 0.2 | 0.1 |
| Glass on ice | 0.1 | 0.03 |
| Glass on glass | 0.95 | 0.4 |
| Metal on metal | 0.6 | 0.4 |
| Lubricated metal on metal | 0.1 | 0.05 |
| Rubber on concrete | 1.0 | 0.8 |
| Wet rubber on concrete | 0.7 | 0.5 |
| Performance tire on concrete | 1.5 | 1.0 |

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## Friction types

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## Friction as impulses

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# How to implement friction?

- We compute the changes of the velocities due to acting impulses.
- Impulsed-based engine.
- Reaction forces are not computed.
- Resting contacts resolved via micro-collisions.
- The standard friction-theory could not be applied.

# Friction as impulses

- What does the friction do in terms of impulses and velocities?

$$\Delta\vec{v} = \Delta\vec{v} - \begin{cases} \vec{v}_{planar} & \text{if } ||\vec{v}_{planar}|| \leq v_c \mu_{static} \\ \vec{v}_{planar}\mu_{dynamic} & \text{else} \end{cases}$$

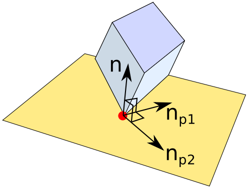


**Figure:** Friction collision basis

- We can compute $\vec{v}_{planar}$ as :

$$\vec{v}_{planar} = \vec{v}_{collision} - \vec{n}.v_c,$$

with $\vec{v}_{collision} = \vec{v}_A - \vec{v}_B$ as defined on slide 16 in Worksheet 6.

# Friction Properties:

## Static friction:

- Keeps the velocity of the object at zero on contact plane.
- Velocity in the direction of the contact normal already adjusted.
- Modify the velocity in the two other directions on the contact plane.

## Dynamic friction:

- Scale the the planar velocity $v_{planar}$ with $\mu_{dynamic}$.

# Angular motion to velocity

## Without friction:

1. Compute the torque per unit impulse in the direction of the contact normal.
2. Convert the torque to rotation using the inertia tensor.
3. Convert the rotation to linear velocity for the current contact point.
4. Convert the velocity back into contact coordinates.

## With friction:

- Change the algorithm so that all three directions of the contact are considered.
- Use the basis matrix $B$ instead of contact normal.
- Compute the maximal impulse $g_{max}$ using:

$$g_{max} = \Delta g v_{\mu},$$

with $v_{\mu}$ being the computed change of velocity due to friction and impulses.

$$\Delta g = \Delta v^{-1}$$

with $v$ the velocity per unit impulse (see next slides).

# The skew-symmetric matrix

In the case of friction we should change the Impulse alogrithm to use all three directions of the contact (one direction is the contact normal and the other two are defined as the contact plane). We need to replace the contact normal with a matrix - the skew matrix.

- Consider $v \times x$ with $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.
- $v \times x = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix} x$
- We can compute $x \times v$ without the matrix form of x because $v \times x = -x \times v$
- We will use the following notation for the skew matrix:

  $S_v$ - the skew matrix for the vector $v$.

# The algorithm for angular motion

1. Compute the impulsive torque $u$ :

$$u = S_{p_{contact}} B$$

with $p_{contact}$ the relative contact position (in object space) and $B$ the transformation matrix to change to world coordinates.

2. Compute the rotation change : torque vector transformed by the tensor matrix. In this case we have simple matrix multiplication.

$$\Delta r = M^{-T} I^{-1} M^T u$$

3. Compute the velocity $v$ per unit impulse.

$$\Delta v_r = \Delta r S_{p_{contact}} (-1)$$

4. Convert into contact coordinates.

$$\Delta v_{r-contact} = B^{-1} \Delta v_r$$

# The algorithm for linear motion

1. Consider the following equation:

$$\Delta v_l = m^{-1} g$$

2. Express the inverse mass as matrix to be added to the angular matrix:

$$\begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & m^{-1} \end{bmatrix}$$