PSE Game Physics

Addendum to Session (1)
Stiffness

Oliver Meister, Sebastian Rettenberger, Roland Wittmann

23.04.2013

Remember this from last session?

- Purely implicit discretisation schemes typically are unconditionally stable

- Explicit schemes are only stable in a certain discretisation range. For time discretisation schemes, the timestep Δt often needs to be limited:

$$\Delta t < C(\text{Problem})$$

But which type is more suitable for a certain phenomena?
$\rightarrow$ consider stiffness!

Stiffness - a simple 1D ODE (1/2)

Testproblem:

- $x'(t) = a \cdot x(t)$ with $a < 0$ (!!) and initial value $x(0) = 1$
- Analytical solution: $x(t) = e^{at}$
- $t \to \infty$: $x(t) \to 0$

What is the best timestep for the explicit Euler method?

- Math says: if we go to an infinite number of timesteps, the result should get very close to zero.
- Like the analytical solution!

First Euler step: $x_{\mathbf{1}} = x_0 + x_0' = x_0 + ax_0 \cdot dt = (1 + a \cdot dt) \cdot x_0$
Second step: $x_{\mathbf{2}} = x_1 + ax_1 \cdot dt = (1 + a \cdot dt)x_1 = (1 + a \cdot dt)^{\mathbf{2}} \cdot x_0$
$\to$ Do you see a pattern and what must hold for $x_{\mathbf{n}} \to 0$?

Stiffness - a simple 1D ODE (2/2)

Pattern: $x_{\mathbf{n}} = (1 + a \cdot dt)^{\mathbf{n}} \cdot x_0$

- Convergence for $|(1 + a \cdot dt)| < 1$ (not 'less or equal'!!)
- Solve for dt and you got the right timestep for the problem!
- $dt = \frac{2}{-a}$ (try it for yourself! not as trivial as it looks)
- if a is large you will need a very small timestep!

Nice toy problem, so what? And how is this related to stiffness?

- Most ODEs have the form $x(t) = A \cdot x'(t)$ with some matrix A
- This matrix (or our scalar) is called the stiffness matrix (value)

Example: shake a simple rod!

Stiffness in a simple rod!

One example: shake a simple rod while keeping one end fixed

- close and open your eyes! has the other end moved?
- if not: close and open your eyes faster! :)
- $\rightarrow$ time stepping

Use your imagination! Try this with a rod made of rubber/steel.

So what type of method is more preferable?

- for large stiffness: implicit methods (not in Game Physics, why?)
- for small stiffness: explicit methods (Game Physics)