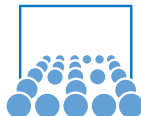


# PSE Game Physics

## Addendum to Session (1) Stiffness

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23.04.2013



Remember this from last session?

- Purely implicit discretisation schemes typically are unconditionally stable
- Explicit schemes are only stable in a certain discretisation range. For time discretisation schemes, the timestep  $\Delta t$  often needs to be limited:

$$\Delta t < C(\text{Problem})$$

But which type is more suitable for a certain phenomena?

→ consider stiffness!

# Stiffness - a simple 1D ODE (1/2)

Testproblem:

- $x'(t) = a \cdot x(t)$  with  $a < 0$  (!! ) and initial value  $x(0) = 1$
- Analytical solution:  $x(t) = e^{at}$
- $t \rightarrow \infty: x(t) \rightarrow 0$

What is the best timestep for the explicit Euler method?

- Math says: if we go to an infinite number of timesteps, the result should get very close to zero.
- Like the analytical solution!

First Euler step:  $x_1 = x_0 + x_0' \cdot dt = x_0 + ax_0 \cdot dt = (1 + a \cdot dt) \cdot x_0$

Second step:  $x_2 = x_1 + ax_1 \cdot dt = (1 + a \cdot dt)x_1 = (1 + a \cdot dt)^2 \cdot x_0$

→ Do you see a pattern and what must hold for  $x_n \rightarrow 0$ ?

## Stiffness - a simple 1D ODE (2/2)

Pattern:  $x_n = (1 + a \cdot dt)^n \cdot x_0$

- Convergence for  $|(1 + a \cdot dt)| < 1$  (not 'less or equal'!!)
- Solve for  $dt$  and you got the right timestep for the problem!
- $dt = \frac{2}{-a}$  (try it for yourself! not as trivial as it looks)
- if  $a$  is large you will need a very small timestep!

Nice toy problem, so what? And how is this related to stiffness?

- Most ODEs have the form  $x(t) = A \cdot x'(t)$  with some matrix  $A$
- This matrix (or our scalar) is called the stiffness matrix (value)

Example: shake a simple rod!

# Stiffness in a simple rod!

One example: shake a simple rod while keeping one end fixed

- close and open your eyes! has the other end moved?
- if not: close and open your eyes faster! :)
- → time stepping

Use your imagination! Try this with a rod made of rubber/steel.

So what type of method is more preferable?

- for large stiffness: implicit methods (not in Game Physics, why?)
- for small stiffness: explicit methods (Game Physics)