PSE Game Physics

Session (4a)
Springs and quaternions extended

Oliver Meister, Sebastian Rettenberger, Roland Wittmann

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Correction 1: Quaternions

Symptom: Mouse rotation of the scene around the vertical axis is inversed.
Reason: There is a bug in the code in main.cpp in cApplicationImplementation::drawFrame():

Replace

```
camera.rotate(-engine.inputState.relative_mouse_y,
    engine.inputState.relative_mouse_x, 0);
```

by

```
camera.rotate(-engine.inputState.relative_mouse_y,
    -engine.inputState.relative_mouse_x, 0);
```

and you should be good.

Correction 2: linear momentum

- Normally, collisions are not completely elastic. The **coefficient of restitution** C_r is used to represent the respective deviation depending on the closing and separating velocity:

$$C_r = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{-v_s}{v_c}$$

- Wrong: By replacing $(v_1 - v_2)$ with the closing velocity v_c and including the coefficient of restitution, the velocity of object 1 after collision is given by

$$\Leftrightarrow v_1' = -2\frac{m_2}{m_1 + m_2} v_c C_r + v_1$$

- Wrong: Similarly, the velocity of object 2 after the collision is given by

$$\Leftrightarrow v_2' = 2\frac{m_1}{m_1 + m_2} v_c C_r + v_2$$

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- Correct: By replacing $(v_1 - v_2)$ with the closing velocity v_c and including the coefficient of restitution, the velocity of object 1 after collision is given by

$$\Leftrightarrow v_1' = -\frac{m_2}{m_1 + m_2} v_c (1 + C_r) + v_1$$

- Correct: Similarly, the velocity of object 2 after the collision is given by

$$\Leftrightarrow v_2' = \frac{m_1}{m_1 + m_2} v_c (1 + C_r) + v_2$$

Undamped springs

We used the formula

$$F(t) = -k\ d(t)$$

to compute the force acting on a spring, where
$d(t)$ = length of spring $l(t)$ - equilibrium length d_0.
Applying this to two objects with mass centers $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^3$, distance $l = |\mathbf{x}_2 - \mathbf{x}_1|$ and masses $m_1, m_2 \in \mathbb{R}^+$ gives:

$$\mathbf{F}_1 = -\mathbf{F}_2 = k\ \frac{l - d_0}{l}(\mathbf{x}_2 - \mathbf{x}_1)$$

This means

$$\mathbf{a}_1 = \frac{\mathbf{F}_1}{m_1} = k\ \frac{l - d_0}{m_1\ l}(\mathbf{x}_2 - \mathbf{x}_1)$$

$$\mathbf{a}_2 = \frac{\mathbf{F}_2}{m_2} = -k\ \frac{l - d_0}{m_2\ l}(\mathbf{x}_2 - \mathbf{x}_1)$$

Damped springs

For damping, we could modify the formula by

$$F(t) = -k\ d(t) - c\ \dot{d}(t)$$

using a *damping coefficient* $c \in \mathbb{R}^+$.
Applying this to two objects with mass centers $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^3$, distance $l = |\mathbf{x}_2 - \mathbf{x}_1|$, velocities $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$ and masses $m_1, m_2 \in \mathbb{R}^+$ gives:

$$\mathbf{F}_1 = -\mathbf{F}_2 = (\frac{k\ (l - d_0)}{l} + \frac{c(\mathbf{v}_2 - \mathbf{v}_1) \cdot (\mathbf{x}_2 - \mathbf{x}_1)}{l^2})(\mathbf{x}_2 - \mathbf{x}_1)$$

Question: Does this method suppress all motion eventually?