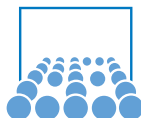


# PSE Game Physics

## Session (4a) Springs and quaternions extended

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## Correction 1: Quaternions

Symptom: Mouse rotation of the scene around the vertical axis is inversed.

Reason: There is a bug in the code in main.cpp in `cApplicationImplementation::drawFrame()`:

Replace

```
camera.rotate(-engine.inputState.relative_mouse_y,  
              engine.inputState.relative_mouse_x, 0);
```

by

```
camera.rotate(-engine.inputState.relative_mouse_y,  
              -engine.inputState.relative_mouse_x, 0);
```

and you should be good.

## Correction 2: linear momentum

- Normally, collisions are not completely elastic. The **coefficient of restitution**  $C_r$  is used to represent the respective deviation depending on the closing and separating velocity:

$$C_r = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{-v_s}{v_c}$$

- **Wrong:** By replacing  $(v_1 - v_2)$  with the closing velocity  $v_c$  and including the coefficient of restitution, the velocity of object 1 after collision is given by

$$\Leftrightarrow v_1' = -2 \frac{m_2}{m_1 + m_2} v_c C_r + v_1$$

- **Wrong:** Similarly, the velocity of object 2 after the collision is given by

$$\Leftrightarrow v_2' = 2 \frac{m_1}{m_1 + m_2} v_c C_r + v_2$$

## Correction 2: linear momentum

- Normally, collisions are not completely elastic. The **coefficient of restitution**  $C_r$  is used to represent the respective deviation depending on the closing and separating velocity:

$$C_r = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{-v_s}{v_c}$$

- Correct: By replacing  $(v_1 - v_2)$  with the closing velocity  $v_c$  and including the coefficient of restitution, the velocity of object 1 after collision is given by

$$\Leftrightarrow v_1' = -\frac{m_2}{m_1 + m_2} v_c (1 + C_r) + v_1$$

- Correct: Similarly, the velocity of object 2 after the collision is given by

$$\Leftrightarrow v_2' = \frac{m_1}{m_1 + m_2} v_c (1 + C_r) + v_2$$

# Undamped springs

We used the formula

$$F(t) = -k d(t)$$

to compute the force acting on a spring, where

$d(t)$  = length of spring  $l(t)$  - equilibrium length  $d_0$ .

Applying this to two objects with mass centers  $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^3$ , distance  $l = |\mathbf{x}_2 - \mathbf{x}_1|$  and masses  $m_1, m_2 \in \mathbb{R}^+$  gives:

$$\mathbf{F}_1 = -\mathbf{F}_2 = k \frac{l - d_0}{l} (\mathbf{x}_2 - \mathbf{x}_1)$$

This means

$$\mathbf{a}_1 = \frac{\mathbf{F}_1}{m_1} = k \frac{l - d_0}{m_1 l} (\mathbf{x}_2 - \mathbf{x}_1)$$

$$\mathbf{a}_2 = \frac{\mathbf{F}_2}{m_2} = -k \frac{l - d_0}{m_2 l} (\mathbf{x}_2 - \mathbf{x}_1)$$

# Damped springs

For damping, we could modify the formula by

$$F(t) = -k d(t) - c \dot{d}(t)$$

using a *damping coefficient*  $c \in \mathbb{R}^+$ .

Applying this to two objects with mass centers  $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^3$ , distance  $l = |\mathbf{x}_2 - \mathbf{x}_1|$ , velocities  $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$  and masses  $m_1, m_2 \in \mathbb{R}^+$  gives:

$$\mathbf{F}_1 = -\mathbf{F}_2 = \left( \frac{k(l - d_0)}{l} + \frac{c(\mathbf{v}_2 - \mathbf{v}_1) \cdot (\mathbf{x}_2 - \mathbf{x}_1)}{l^2} \right) (\mathbf{x}_2 - \mathbf{x}_1)$$

Question: Does this method suppress all motion eventually?