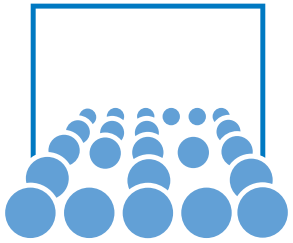
PSE Game Physics

Session (2)
Collisions: Sphere-Sphere, Sphere-Plane Interpenetrations

Oliver Meister, Roland Wittmann

25.04.2014



Outline

Some very very basic math

Linear transformations

Collisions

- Sphere-Sphere
- Collision Normal
- Collision points & Interpenetration depth

Outline

Some very very basic math

Linear transformations

Collisions

- Sphere-Sphere
- Collision Normal
- Collision points & Interpenetration depth

Right-handed system

- Our **SBND-Engine** is written in a **Right-handed system** (OpenGL):

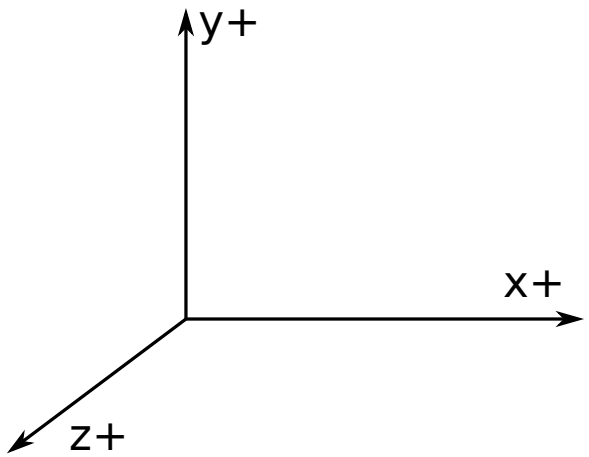
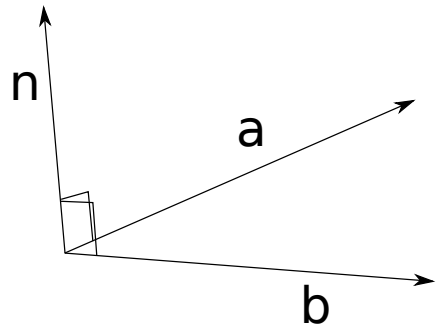


Figure: right-handed coordinate system

- Use your hand to determine the orientation: Thumb (X-axis), forefinger (Y-axis), middlefinger (Z-axis).
- Thus moving an object further away means **decreasing** the z-coordinate!
- All formulae are given in left handed basis!

Cross product



$$\vec{n} = \vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

- We mainly use it to compute a vector $\vec{n}$ which is perpendicular to two given vectors $\vec{a}$ and $\vec{b}$
- E. g. if a plane is given by a single point and 2 vectors, the **plane normal** can be computed
- Remember: This formula is given in a **right-handed coordinate system**!

Dot product

$$\vec{a} \cdot \vec{b} = \sum_{i=1}^{3} a_i b_i$$

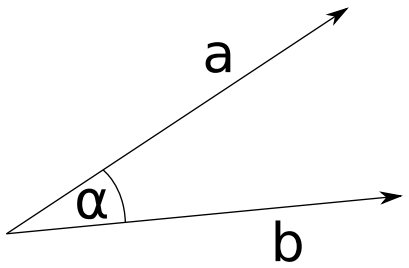


Figure: dot product

- Usage: **Compute the cosine** of the angle created by $\vec{a}$ and $\vec{b}$:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos(\alpha)$$

- Later on, we use it to compute the **projected length** of $\vec{a}$ on a normal $\vec{n}$

Outline

Some very very basic math

Linear transformations

Collisions

- Sphere-Sphere
- Collision Normal
- Collision points & Interpenetration depth

Object properties

Video: NaturalMotion by Euphoria

http://www.youtube.com/watch?v=Qi5adyccoKI

Object properties

- Position
- Orientation
- Velocity
- Mass
- ...

How can we express e. g. Position and Orientation?

Degrees of Freedom for a rigid body object

”Modifications” of Position and Orientation which we need for our physics engine with **rigid bodies**:

- **Translation**: Move object from position $p(t)$ to $p(t + \Delta t)$
- **Orientation**: Rotate an object $q(t)$ to $q(t + \Delta t)$

Other Modifications not necessary for our physics engine:

- **Shearing** (Meaningless for rigid body objects)
- **Scaling** (Meaningless for rigid body objects)
- **Projections** (Only necessary for visualization, non-affine transformation)

How to describe the transformations?

Model- and World-space:

- **Model-Space**: The *space* in which the object is setup without any modifications.
- The object is **moved/rotated** during initialization or by applying a linear or rotational velocity in each timestep.
- Applying this transformation projects the object **from model-space to world-space**.

Linear transformations:

- Store the *position* of the object in a **vector** $\vec{p}$.
- Store the *orientation* of the object in a **quaternion** $\vec{q}$.
- Use a 4×4 matrix M which is created based on the position and quaternion to describe the transformation.
- (for most cases, only a 4×3 matrix is necessary)

Translation

- A translation can be expressed by setting specific values in the last column of our 4*x*4 matrix *M*:

$$M_{translate} = \begin{pmatrix} 1 & & & t_x \\ & 1 & & t_y \\ & & 1 & t_z \\ & & & 1 \end{pmatrix}$$

- To allow a translation simply by applying the matrix *M* to a postion, we extend the objects position $\vec{p}$ by a fourth component which is set to 1 to the **homogenous vector** $\vec{\vec{p}}$.
- Example: $\vec{\vec{p}} = (1, 3, -2, 1)^T$, $t_x = (10, 10, 10)^T$

$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & . & . & t_x \\ . & 1 & . & t_y \\ . & . & 1 & t_z \\ . & . & . & 1 \end{pmatrix} \cdot \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix} = \begin{pmatrix} p_x + t_x \\ p_y + t_y \\ p_z + t_z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + t_x \\ 3 + t_y \\ -2 + t_z \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 13 \\ 8 \\ 1 \end{pmatrix}$$

Orientation

- One way to express 3D Orientation is to combine 2D rotations.
- E. g. 2D rotation around Z-Axis:

$$M_{rotate_Z} = \begin{pmatrix} \cos\alpha & -\sin\alpha & . & . \\ \sin\alpha & \cos\alpha & . & . \\ . & . & 1 & . \\ . & . & . & 1 \end{pmatrix}$$

- Combining 3 rotation matrices (Around X, Y and Z axis), we can express all possible rotations.

Putting everything together

- Now we can express rigid body transformations by **applying all transformations to the object's position** (or at any point on the object's surface:

$$M_{translate} \cdot M_{rotate} \cdot \vec{p}$$

- By setting $M = M_{translate} \cdot M_{rotate}$ the transformation can be expressed by only a **single matrix-vector expression**:

$$M \cdot \vec{p}$$

- Model matrix M gives the transformation of an object **from mode-space to world-space**.
- (Model matrix is also called world matrix when programming with DirectX).

Inverse Transformations

- It's also possible to **transform objects from world-space to the model-space**.
- This becomes handy when **intersection tests** can be easily **handled in the model-space** of an object (e. g. ball-box intersection tests)
- The transformation of an object O_1 to the model-space of another object O_2 can be simply handled by using the inverse of the **model-matrix**:

$$M_2^{-1} \cdot M_1$$

- Hint for a better understanding: **read this formula from right to left**
  - Firstly, object 1 is transformed to the world space
  - By applying the inverse transformation for object 2, transforms object 1 to the model-space of object 2

How to transform Vectors?

- A **vector** at an arbitrary position **cannot be transformed simply by applying the model-matrix**!!!
- To transform a vector $\vec{v}$ from one space to another given the point transformation matrix M, the vector $\vec{v}$ can be either transformed by
  - Applying M to the origin point as well as to the vector $\vec{v}$ and subtracting both values to get the appropriate solution or
  - by applying the **inverse transpose** of M (see next slide)

Inverse-transpose of a matrix (1/2)

- Here we give a short sketch (See `http://www.faqs.org/faqs/graphics/algorithms-faq/` for more details)
- We start by **splitting up** the matrix M into the 3x3 **matrix** L and the **translation vector** $\vec{t}$:

$$M = \left(L \mid \vec{t} \right)$$

- Assuming that $\vec{p}$ and $\vec{q}$ lie on an object's surface, the vector connecting both points is defined by $\vec{v} = \vec{p} - \vec{q}$.
- Then we can describe the transformation of an arbitrary vector $\vec{v}$ connecting with the transformation of its starting and end point $\vec{p}$ and $\vec{q}$:

$$\vec{v}' = (L\vec{p} + \vec{t}) - (L\vec{q} + \vec{t}) = L(\vec{p} - \vec{q}) = L(\vec{v})$$

Inverse-transpose of a matrix (2/2)

- When transforming two vectors, the **angle remains the same** which can be expressed by a **dot-product**.
- In this case, our second vector is represented by a **normal** $\vec{n}$ on the **surface point** $\vec{q}$.
- Since we know, that $\vec{v}$ can be transformed by applying L, we can rewrite the equation to

$$\vec{n}' \cdot \vec{v}' = (N\vec{n}) \cdot (L\vec{v})$$

  with N being the matrix we like to figure out.
- By replacing the dot product using the transposed:

$$\vec{n}' \cdot \vec{v}' = (N\vec{n})^T (L\vec{v}) = \vec{n}^T N^T L\vec{v} = \vec{n}'^T \vec{v}'$$

- Since the angle between the 2 vectors $\vec{n}$ and $\vec{v}$ also **have to be equal in model-space**, we can assume that $N^T L = I$.
- Thus, we get $N = L^{-T}$ for the matrix **transforming vectors** to the basis given in M.

Overview

- So far we are able to **mathematically describe transformations from model- to world-space**.
- Use homogenous coordinates $(x_1, x_2, x_3, 1)$ for **translations** and position.
- Use homogenous coordinates $(x_1, x_2, x_3, 0)$ for **vectors**.
- Use **transposed inverse matrices M^{-T} to transform vectors**.
- Use **inverse** M^{-1} of transformation matrix to **transform "backward"**.

Overview

- So far we are able to **mathematically describe transformations from model- to world-space**.
- Use homogenous coordinates $(x_1, x_2, x_3, 1)$ for **translations** and position.
- Use homogenous coordinates $(x_1, x_2, x_3, 0)$ for **vectors**.
- Use **transposed inverse matrices M^{-T} to transform vectors**.
- Use **inverse** M^{-1} of transformation matrix to **transform "backward"**.
- Seems to be easy, isn't it? Not for rotations!

Outline

Some very very basic math

Linear transformations

Collisions

- Sphere-Sphere
- Collision Normal
- Collision points & Interpenetration depth

Our schedule for collision tests

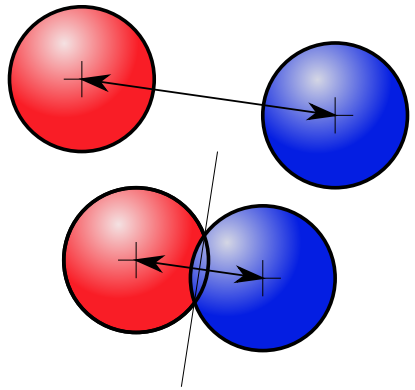
- Collision tests:

	Plane	Sphere	Box
Plane	o	-	-
Sphere	a	X	-
Box	a	X	X

  - o: meaningless
  - a: not in this session, but available via additional slides.
  - -: Symmetric
- Resolving interpenetration

Collision: Sphere-Sphere

- Really easy to detect this collision:
  Test whether the **distance of the 2 center points is less than the sum of both radii**
- But we need more collision data in order to handle collisions...
  (next slide)



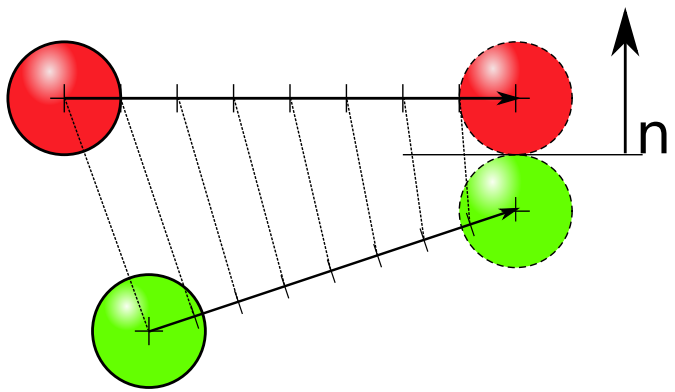
Intersection data

Which intersection data has to be generated generated?

- Collision normal
- Collision points
- Interpenetration depth

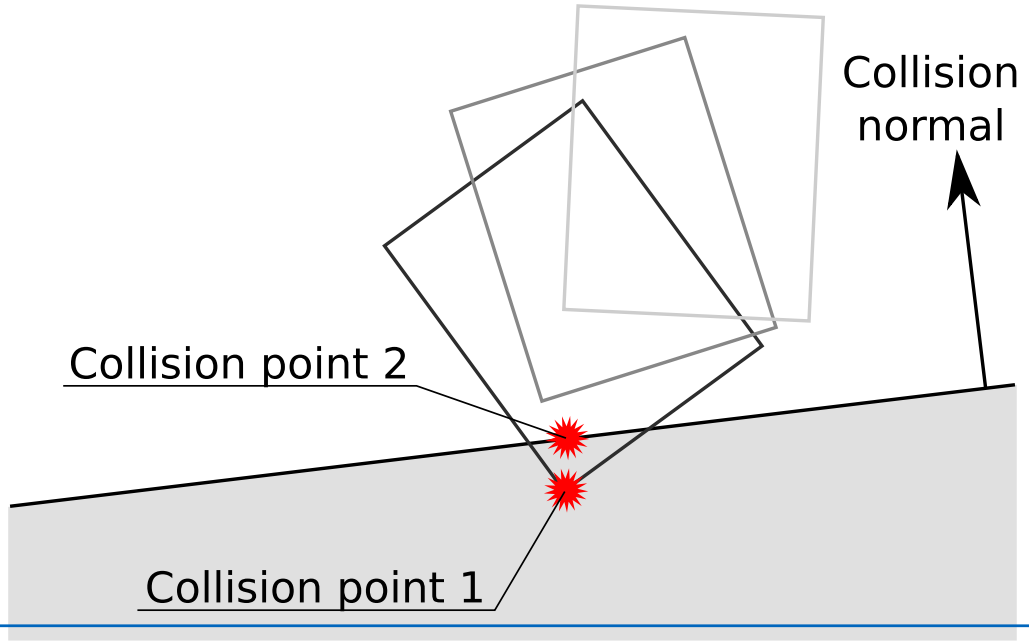
Collision Normal

- We need to know in which **direction** we have to **apply some force** due to the collision.
- For spheres we assume to determine the collision normal when the 2 spheres are very close.
- Thus the normal from the first sphere aiming to the other one is given by computing the **collision normal** based upon **both sphere's centers**.



Collision points & Interpenetration depth

- Since an interpenetration can occur during a timestep (not only at the very end), the collision point for every object has to be approximated.
- This collision point gets important when **applying an impulse** to an object which results in a angular momentum.
- The distance of those collision points also gives us the **interpenetration depth**.



Resolving Interpenetration (1/2)

- If a **binary-collision for 2 objects was detected**, this interpenetration has to be solved somehow
- The first assumption is, that the interpenetration can be solved by moving the objects **parallel to the collision normal**
- Question: **How far** should every object be moved?

Resolving Interpenetration (2/2)

- We start by considering **2 extreme cases** assuming that resolving the interpenetration depends on the rigid-bodys mass
  - O1 has ∞ mass, O2 has mass 1 $\Rightarrow$ Only O2 is moved
  - O2 has ∞ mass, O1 has mass 1 $\Rightarrow$ Only O1 is moved
- Linearly interpolating inbetween these 2 extreme cases, the fraction d_1 of the interpenetration distance for O1 can be computed by:

$$d_1 = \frac{m_2}{m_1 + m_2} = \frac{m_1^{-1}}{m_1^{-1} + m_2^{-1}}$$

with m_i^{-1} being the inverse mass of the corresponding objects.