PSE Game Physics

Session (7)
Friction

Oliver Meister, Roland Wittmann

12.06.2015

Outline

Angular Momentum Extended

Vocabulary

Friction types

- Static friction
- Dynamic friction
- Some coefficients

Friction as impulses

- Change of velocity due to friction
- The modified velocity resolution algorithm

Collision resolution summary

Steps to resolve a collision:

Collision resolution summary

Steps to resolve a collision:

1. Choose as (initially wrong) impulse the unit vector $\vec{n}$ (which is the collision normal).

Collision resolution summary

Steps to resolve a collision:

1. Choose as (initially wrong) impulse the unit vector $\vec{n}$ (which is the collision normal).
2. For each object, compute the change of linear and rotation velocity $\Delta\vec{v}_l, \Delta\vec{v}_r$ in the collision point, that would occur if we applied the unit impulse $\vec{n}$ (p. 18)

Collision resolution summary

Steps to resolve a collision:

1. Choose as (initially wrong) impulse the unit vector $\vec{n}$ (which is the collision normal).
2. For each object, compute the change of linear and rotation velocity $\Delta\vec{v}_l, \Delta\vec{v}_r$ in the collision point, that would occur if we applied the unit impulse $\vec{n}$ (p. 18)
3. For each object, compute the total velocity change $\Delta\vec{v}$ at the collision point by adding up change of linear velocity and cross product of change of rotation velocity and lever arm (p. 18).

Collision resolution summary

Steps to resolve a collision:

1. Choose as (initially wrong) impulse the unit vector $\vec{n}$ (which is the collision normal).
2. For each object, compute the change of linear and rotation velocity $\Delta\vec{v}_l, \Delta\vec{v}_r$ in the collision point, that would occur if we applied the unit impulse $\vec{n}$ (p. 18)
3. For each object, compute the total velocity change $\Delta\vec{v}$ at the collision point by adding up change of linear velocity and cross product of change of rotation velocity and lever arm (p. 18).

Applying the unit impulse to two colliding objects, we would get the separating velocity $v_s = v_c + (\Delta\vec{v}_1 - \Delta\vec{v}_2) \cdot \vec{n}$. v_s does not fulfill the condition $v_s = -C_r \cdot v_c$, however.

Collision resolution summary

Steps to resolve a collision:

1. Choose as (initially wrong) impulse the unit vector $\vec{n}$ (which is the collision normal).
2. For each object, compute the change of linear and rotation velocity $\Delta\vec{v}_l, \Delta\vec{v}_r$ in the collision point, that would occur if we applied the unit impulse $\vec{n}$ (p. 18)
3. For each object, compute the total velocity change $\Delta\vec{v}$ at the collision point by adding up change of linear velocity and cross product of change of rotation velocity and lever arm (p. 18).

Applying the unit impulse to two colliding objects, we would get the separating velocity $v_s = v_c + (\Delta\vec{v}_1 - \Delta\vec{v}_2) \cdot \vec{n}$. v_s does not fulfill the condition $v_s = -C_r \cdot v_c$, however.

4. So in order to fulfill the condition, we have to multiply the unit impulse with a scalar fraction f. Since all performed operations are linear, we get $-C_r \cdot v_c = v_s = v_c + f \cdot (\Delta\vec{v}_1 - \Delta\vec{v}_2) \cdot \vec{n}$.

Collision resolution summary

Steps to resolve a collision:

1. Choose as (initially wrong) impulse the unit vector $\vec{n}$ (which is the collision normal).
2. For each object, compute the change of linear and rotation velocity $\Delta\vec{v}_l, \Delta\vec{v}_r$ in the collision point, that would occur if we applied the unit impulse $\vec{n}$ (p. 18)
3. For each object, compute the total velocity change $\Delta\vec{v}$ at the collision point by adding up change of linear velocity and cross product of change of rotation velocity and lever arm (p. 18).

Applying the unit impulse to two colliding objects, we would get the separating velocity $v_s = v_c + (\Delta\vec{v}_1 - \Delta\vec{v}_2) \cdot \vec{n}$. v_s does not fulfill the condition $v_s = -C_r \cdot v_c$, however.

4. So in order to fulfill the condition, we have to multiply the unit impulse with a scalar fraction f. Since all performed operations are linear, we get $-C_r \cdot v_c = v_s = v_c + f \cdot (\Delta\vec{v}_1 - \Delta\vec{v}_2) \cdot \vec{n}$.
5. Solve the equation by f and set $\vec{v}_l := \vec{v}_l + f \cdot \Delta\vec{v}_l$, $\vec{v}_r := \vec{v}_r + f \cdot \Delta\vec{v}_r$.

Debugging the collision resolution

There are physical conditions for the collision resolution, which we can exploit for debugging:

Debugging the collision resolution

There are physical conditions for the collision resolution, which we can exploit for debugging:

- Separating velocity: After the collision $v_s = -C_r \cdot v_c$.

Debugging the collision resolution

There are physical conditions for the collision resolution, which we can exploit for debugging:

- Separating velocity: After the collision $v_s = -C_r \cdot v_c$.
- Momentum conservation: The total linear momentum of the two objects $\sum \vec{p}_l$ is invariant. Likewise, total angular momentum $\sum \vec{p}_r$. Remark: angular momentum is conserved only for momentums w.r.t to the same reference point. This point may be chosen freely, but it must be equal for both objects.

Debugging the collision resolution

There are physical conditions for the collision resolution, which we can exploit for debugging:

- Separating velocity: After the collision $v_s = -C_r \cdot v_c$.
- Momentum conservation: The total linear momentum of the two objects $\sum \vec{p}_l$ is invariant. Likewise, total angular momentum $\sum \vec{p}_r$. Remark: angular momentum is conserved only for momentums w.r.t to the same reference point. This point may be chosen freely, but it must be equal for both objects.
- Energy conservation: If $C_r = 1$, the total kinetic energy is invariant, where $E_{kin} = \sum \frac{1}{2} m \vec{v}_l^2 + \frac{1}{2} \vec{v}_r^T I \vec{v}_r$. If $C_r < 1$, E_{kin} decreases.

Debugging the collision resolution

There are physical conditions for the collision resolution, which we can exploit for debugging:

- Separating velocity: After the collision $v_s = -C_r \cdot v_c$.
- Momentum conservation: The total linear momentum of the two objects $\sum \vec{p}_l$ is invariant. Likewise, total angular momentum $\sum \vec{p}_r$. Remark: angular momentum is conserved only for momentums w.r.t to the same reference point. This point may be chosen freely, but it must be equal for both objects.
- Energy conservation: If $C_r = 1$, the total kinetic energy is invariant, where $E_{kin} = \sum \frac{1}{2} m \vec{v}_l^2 + \frac{1}{2} \vec{v}_r^T I \vec{v}_r$. If $C_r < 1$, E_{kin} decreases.

$\rightarrow$ Use your knowledge of the model!

Outline

Vocabulary

Linear Mechanics		Angular Mechanics	
English	German	English	German
Acceleration	Beschleunigung	Angular Acceleration	Winkelbeschleunigung
Velocity	Geschwindigkeit	Angular Velocity	Winkelgeschwindigkeit
Position	Position	Orientation	Orientierung
Mass	Masse	Inertia Tensor	Trägheitstensor
Force	Kraft	Torque	Drehmoment
Momentum	Impuls	Angular Momentum	Drehimpuls
Impulse	Kraftstoß	Angular Impulse	Drehmomentstoß
Static Friction	Haftreibung		
Dynamic Friction	Gleitreibung		

Outline

Angular Momentum Extended

Vocabulary

Friction types

- Static friction
- Dynamic friction
- Some coefficients

Friction as impulses

- Change of velocity due to friction
- The modified velocity resolution algorithm

Friction

- A **force** generated when one object moves or tries to move in contact with another object.
- Two types of friction: **static** and **dynamic** friction.
- Results into **resistance** at the point of contact to the current motion.

Static friction - Part 1

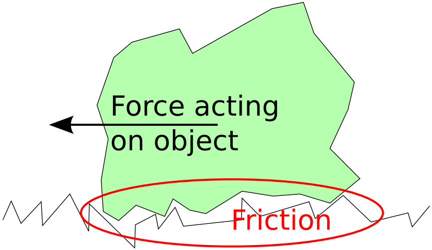

Figure: Example for static friction between an object and a surface

Occurs when colliding objects are interlocked at their contact points (e.g. imagine two gears). Any relative movement at the contact point is prevented, since force created by static friction stops it from moving.

Static friction - Part 2

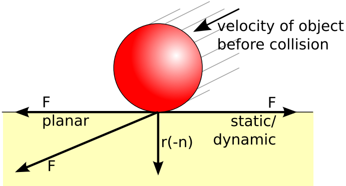

Figure: Example for static friction between sphere and plane

- The **reaction force** $\vec{r}$ is calculated:

$$\vec{r} = -|\vec{f} \cdot \vec{n}|\vec{n},$$

  with $\vec{f}$ being the force acting between both objects.
- Static friction force depends on the material at the point of contact (the coefficient μ) and the reaction force $\vec{r}$:

$$|\vec{f}_{static}| \leq \mu_{static}|\vec{r}|$$

Static friction - Part 3

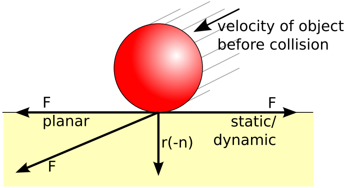

Figure: Example for static friction between sphere and plane

- μ_{static} an empirical property described in tables for different pairs of materials.
- Compute $\vec{f}_{planar}$, e. g. using $\vec{f}_{planar} = \vec{f} - \vec{r}$
- If a static friction is detected ($|\vec{f}_{planar}| \leq \mu_{static}|\vec{r}|$), use $\vec{f}_{static} = -\vec{f}_{planar}$

Dynamic friction - Part 1

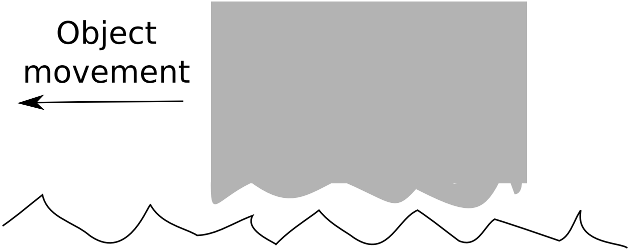

Figure: Example for dynamic friction between an object and a surface

- Has similar behaviour as the static friction.
- Described through different friction coefficient $\mu_{dynamic}$.
- Applied, once the static friction force is exceeded.

Dynamic friction - Part 2

- The equation for the dynamic friction depends on the dynamic friction coefficient $\mu_{dynamic}$ as well as the reaction force $\vec{r}$:

$$|\vec{f}_{dynamic}| = -\mu_{dynamic}|\vec{r}|$$

- Similar to static friction, this force acts parallel to the planar force:

$$\vec{f}_{dynamic} = -\mu_{dynamic}\frac{|\vec{r}|}{|f_{planar}|}f_{planar}$$

Some coefficients

Materials	Static friction	Dynamic friction
Wood on stone	0.7	0.3
steel on steel	0.16	0.12

Outline

How to implement friction?

- **Impulse-based** engine.
- We compute the changes of the velocities due to acting impulses.
- Reaction forces are not computed.
- The standard friction theory cannot be applied directly.

Friction Properties

Friction Properties:

Static friction:

- Set the planar velocity v_{planar} to zero.
- Velocity in the direction of the contact normal already adjusted.
- Modify the velocity in the two other directions on the contact plane.

Dynamic friction:

- Scale the planar velocity v_{planar} with $\mu_{dynamic}$.

(Test: Assert that v_{planar} is set correctly after the collision)

Friction as impulses

- What does the friction do in terms of impulses and velocities?

$$\Delta\vec{v} = \Delta\vec{v} - \begin{cases} \vec{v}_{planar} & \text{if } ||\vec{v}_{planar}|| \leq v_c \cdot \mu_{static} \\ \mu_{dynamic} \cdot \vec{v}_{planar} & \text{else} \end{cases}$$

- Here we approximate the dynamic change of velocity using the planar velocity instead of the closing velocity directly.
- We can compute $\vec{v}_{planar}$ as :

$$\vec{v}_{planar} = \vec{v}_{collision} - v_c \cdot \vec{n}$$

with $\vec{v}_{collision} = \vec{v}_A - \vec{v}_B$ as defined on slide 16 in presentation slides of session 6.

Angular motion to velocity

Without friction:

1. Compute the **torque per unit impulse** in the direction of the contact normal.
2. Convert the **torque to rotation** using the inertia tensor.
3. Convert the **rotation to linear velocity** for the current contact point.
4. Convert the **velocity** back into **contact coordinates**

Finally we have the change of velocity per unit impulse aligned at collision normal.

With friction:

- Change the algorithm so that all **three directions of the contact are considered**.
- Use the basis matrix B instead of contact normal.
- Our new task: Compute the impulse g_{μ} which has to be applied to gain the desired change of velocity for handling
  1. angular momentum
  2. friction forces

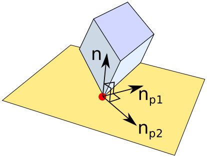

Figure: Friction collision basis

The skew-symmetric matrix

In the case of friction we should change the Impulse algorithm to use all three directions of the contact (one direction is the contact normal and the other two are defined as the contact plane). We need to replace the contact normal with a matrix - the skew matrix.

- Consider $\vec{v} \times \vec{w}$ with $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.
- Using the skew matrix, we can rewrite this equation to

$$\vec{v} \times \vec{w} = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix} \vec{w}$$

- Keep in mind that $\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$ in case that the skew matrix of $\vec{w}$ is already computed!
- We will use the following notation for the skew matrix: $S_{\vec{v}}$ - the skew matrix for the vector $\vec{v}$.

The algorithm for linear motion

Question: How to handle scalar values?

1. Consider the following equation:

$$\vec{g}_\mu = m \cdot \Delta\vec{v}_l \Rightarrow \Delta\vec{v}_l = m^{-1} \cdot \vec{g}_\mu$$

2. Express the inverse mass as matrix to be added to the angular matrix:

$$\begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & m^{-1} \end{bmatrix}$$

The algorithm for angular motion (1/2)

1. Compute angular impulses $\Delta\mathbf{U}$ for each object when applying unit impulses:

$$\Delta\mathbf{U} = \mathbf{X} \cdot \mathbf{N}$$

with $\mathbf{X}$ representing the lever arm equal to $\mathbf{S}_{\vec{p}_{contact}}$ with $\vec{p}_{contact}$ the relative contact position (in each object space). $\mathbf{N}$ represents the orthonormal basis for the collision space with one axis representing the collision normal.

2. Compute the angular velocity changes: angular impulses transformed by inertia tensor matrix. In this case we have a simple matrix-multiplication:

$$\Delta\mathbf{V}_r = \mathbf{M}^{-T}\mathbf{I}^{-1}\mathbf{M}^T\Delta\mathbf{U}$$

(Hint: Compare this with the formulae for the angular momentum!)

The algorithm for angular motion (2/2)

4. Compute the linear velocity change $\Delta\mathbf{V}$ per unit impulses at collision point.
$$\Delta\mathbf{V} = \Delta\mathbf{V}_r\mathbf{X} + \Delta\mathbf{V}_l$$
5. Convert velocity from collision coordinates into world coordinates:
$$\Delta\mathbf{V}_{\text{contact}} = \mathbf{N}^{-1}\Delta\mathbf{V}$$
6. Finally, impulses have to be applied to fit our friction model (e. g. removing all planar velocity changes for static friction). This leads to the following system of equations with $\vec{v}_{change}$ representing the desired change in velocity in collision space and $\vec{p}$ the desired impulse to be applied:
$$\left(\Delta\mathbf{V}_{\text{contact}}\right)\left(\vec{p}\right) = \left(\vec{v}_{change}\right)$$