FSI in a nutshell

FSI Seminar

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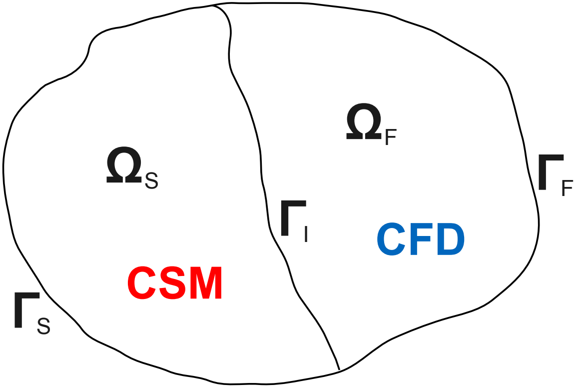
Domain

$$\Omega = \Omega_F \,\dot{\cup}\, \Omega_S,\ \Gamma_I = \Gamma_F \cap \Gamma_S$$



Navier-Stokes Equations

…for incompressible flow

$$\begin{aligned} \partial_t(\rho \mathbf{u}) - \mu\Delta\mathbf{u} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p &= \rho\,\mathbf{g} \quad \text{in } \Omega_F \\ \nabla\cdot\mathbf{u} &= 0 \quad \text{in } \Omega_F \\ +\,\text{BC} & \qquad \text{on } \Gamma_F \end{aligned}$$

Topic 1

- more details
- deduction of the equation
- weak form
- how to calculate forces on obstacles
- …

Discretization, Topic 2

FE, FV or FD $\Rightarrow$ we will focus on FE

$$
\begin{aligned}
\mathbf{M} \cdot \partial_t U + \mathbf{C}(U) \cdot U + \mathbf{A} \cdot U + \mathbf{B}^T P &= G \\
\mathbf{B} U &= 0 \\
+ \text{BC}
\end{aligned}
$$

- time discretization: FE, BE, CN, ...
- decoupling
- Eulerian vs. ALE approach
- balanced FE spaces vs. p-stabilization
- spatial resolution vs. u-stabilization
- timestep size limitation, CFL-condition
- ...

Structural Mechanics, Topic 3

$$\begin{aligned} \rho\, \partial_{tt} \mathbf{d} &= \nabla \cdot \mathbf{S}(\mathbf{d}) + \rho\, \mathbf{f} \quad \text{in } \Omega_S \\ &+ \text{ BC} \qquad \text{on } \Gamma_S \end{aligned}$$

- deduction of the equation
- models for **S** (2nd Pinola-Kirchhoff stress tensor)
- Lagrangian point of view
- discretization with FE
- ...

Coupling Equations

kinematic interface conditions

$$\mathbf{x}_F = \mathbf{d}_S \text{ and } \mathbf{u}_F = \partial_t \mathbf{d}_\mathbf{S} \text{ on } \Gamma_I$$

dynamic interface condition

$$\sigma_F \cdot \mathbf{n}_F = -\sigma_S \cdot \mathbf{n}_S \text{ on } \Gamma_I$$

Monolithic Approach, Topic 4

equations: N_i, unknowns: d_i
for $i = 1$ (Fluid), $i = 2$ (Structure), $i = 3$ (Mesh)

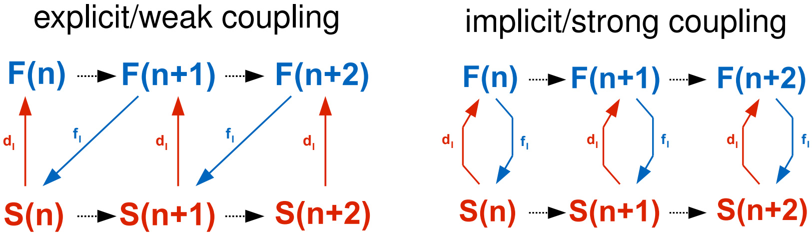
$$N_1(d_1, d_2, d_3) = F_1$$
$$N_2(d_1, d_2, d_3) = F_2$$
$$N_3(d_1, d_2, d_3) = F_3$$

solution: e.g. direct coupling

$$\begin{pmatrix} \frac{\partial N_1}{\partial d_1} & \frac{\partial N_1}{\partial d_2} & \frac{\partial N_1}{\partial d_3} \\ \frac{\partial N_2}{\partial d_1} & \frac{\partial N_2}{\partial d_2} & \frac{\partial N_2}{\partial d_3} \\ \frac{\partial N_3}{\partial d_1} & \frac{\partial N_3}{\partial d_2} & \frac{\partial N_3}{\partial d_3} \end{pmatrix} \begin{pmatrix} \Delta d_1^i \\ \Delta d_2^i \\ \Delta d_3^i \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} - \begin{pmatrix} N_1(d_1^i, d_2^i, d_3^i) \\ N_2(d_1^i, d_2^i, d_3^i) \\ N_3(d_1^i, d_2^i, d_3^i) \end{pmatrix}$$

Partitioned Approach, Topic 5

- motivation: flexibilty
- abstract notation: $F : \mathbf{d}_I \mapsto \mathbf{f}_I$, $S : \mathbf{f}_I \mapsto \mathbf{d}_I$
- stability issues $\Rightarrow$ explicit coupling or implicit coupling with Gauß-Seidel scheme does not work (for incompressible flow)
- solutions: underrelexation, interface quasi-Newton, ...

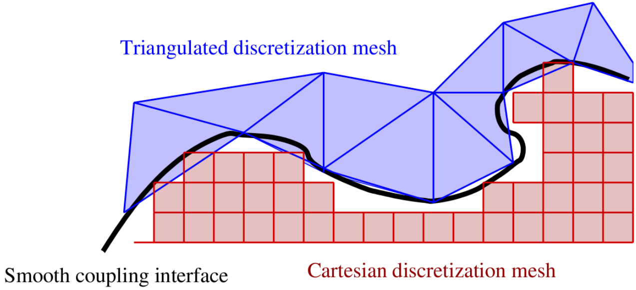


Added-Mass Effect, Topic 6

- instability for the partitioned approach
- structure solver does not “see” the fluid, hence the added-mass
- a problem for incompressible flow
- gets worse for lower mass ratio m_S / m_{added}

Data mapping for non-matching grids, Topic 7

Force, momentum and energy conservation is important ⇒ not trivial



Software

- COMSOL - commercial CSM tool (Topic 8)
- Peano - inhouse fluid solver (Topic 9)
- preCICE - inhouse coupling environment (Topic 10)