FSI in a nutshell

FSI Seminar

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Domain

\[ \Omega = \Omega_F \cup \Omega_S, \Gamma_I = \Gamma_F \cap \Gamma_S \]
Navier-Stokes Equations

... for incompressible flow

\[
\partial_t (\rho u) - \mu \Delta u + \rho (u \cdot \nabla) u + \nabla \rho = \rho g \quad \text{in } \Omega_F \\
\nabla \cdot u = 0 \quad \text{in } \Omega_F \\
+ \text{BC} \quad \text{on } \Gamma_F
\]

Topic 1

- more details
- deduction of the equation
- weak form
- how to calculate forces on obstacles
- ...
Discretization, Topic 2

FE, FV or FD ⇒ we will focus on FE

\[ M \cdot \partial_t U + C(U) \cdot U + A \cdot U + B^T P = G \]
\[ BU = 0 \]
\[ + BC \]

- time discretization: FE, BE, CN, . . .
- decoupling
- Eulerian vs. ALE approach
- balanced FE spaces vs. p-stabilization
- spatial resolution vs. u-stabilization
- timestep size limitation, CFL-condition
- . . .
\[ \rho \partial_{tt} \mathbf{d} = \nabla \cdot \mathbf{S}(\mathbf{d}) + \rho \mathbf{f} \quad \text{in } \Omega_S \\
+ \quad \text{BC on } \Gamma_S \]

- deduction of the equation
- models for \( \mathbf{S} \) (2nd Pinola-Kirchhoff stress tensor)
- Lagrangian point of view
- discretization with FE
- \ldots
Coupling Equations

kinematic interface conditions

\[ x_F = d_S \quad \text{and} \quad u_F = \partial_t d_S \quad \text{on} \ \Gamma_I \]

dynamic interface condition

\[ \sigma_F \cdot n_F = -\sigma_S \cdot n_S \quad \text{on} \ \Gamma_I \]
Monolithic Approach, Topic 4

equations: \( N_i \), unknowns: \( d_i \)
for \( i = 1 \) (Fluid), \( i = 2 \) (Structure), \( i = 3 \) (Mesh)

\[
N_1(d_1, d_2, d_3) = F_1 \\
N_2(d_1, d_2, d_3) = F_2 \\
N_3(d_1, d_2, d_3) = F_3
\]

solution: e.g. direct coupling

\[
\begin{bmatrix}
\frac{\partial N_1}{\partial d_1} & \frac{\partial N_1}{\partial d_2} & \frac{\partial N_1}{\partial d_3} \\
\frac{\partial N_2}{\partial d_1} & \frac{\partial N_2}{\partial d_2} & \frac{\partial N_2}{\partial d_3} \\
\frac{\partial N_3}{\partial d_1} & \frac{\partial N_3}{\partial d_2} & \frac{\partial N_3}{\partial d_3}
\end{bmatrix}
\begin{bmatrix}
\Delta d_1^i \\
\Delta d_2^i \\
\Delta d_3^i
\end{bmatrix}
= 
\begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix}
- 
\begin{bmatrix}
N_1(d_1^i, d_2^i, d_3^i) \\
N_2(d_1^i, d_2^i, d_3^i) \\
N_3(d_1^i, d_2^i, d_3^i)
\end{bmatrix}
\]
Partitioned Approach, Topic 5

- motivation: flexibility
- abstract notation: $F : d_I \mapsto f_I$, $S : f_I \mapsto d_I$
- stability issues $\Rightarrow$ explicit coupling or implicit coupling with Gauß-Seidel scheme does not work (for incompressible flow)
- solutions: underrelaxation, interface quasi-Newton, ...

### explicit/weak coupling

$$
\begin{align*}
F(n) &\rightarrow F(n+1) &\rightarrow F(n+2) \\
S(n) &\rightarrow S(n+1) &\rightarrow S(n+2)
\end{align*}
$$

### implicit/strong coupling

$$
\begin{align*}
F(n) &\rightarrow F(n+1) &\rightarrow F(n+2) \\
S(n) &\rightarrow S(n+1) &\rightarrow S(n+2)
\end{align*}
$$
Added-Mass Effect, Topic 6

- instability for the partitioned approach
- structure solver does not “see” the fluid, hence the added-mass
- a problem for incompressible flow
- gets worse for lower mass ratio $m_S/m_{added}$
Data mapping for non-matching grids, **Topic 7**

Force, momentum and energy conservation is important ⇒ not trivial

Triangulated discretization mesh

Smooth coupling interface

Cartesian discretization mesh
Software

- COMSOL - commercial CSM tool (Topic 8)
- Peano - inhouse fluid solver (Topic 9)
- preCICE - inhouse coupling environment (Topic 10)