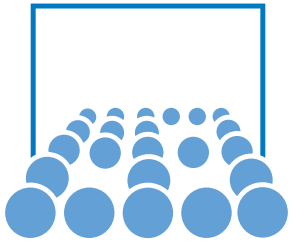
Preliminary Meeting

Seminar Multigrid Methods

Benjamin Peherstorfer, Marion Weinzierl

January 22, 2012



Organisational Issues

- Seminar will be in English
- Preliminary meeting: now!
- Kick-off with final assignment of topics: first semester week
- Working on topics during the semester break possible

→ You can chose your topic now

Requirements

- Solid understanding of your topic
- Optional: Implementation of underlying algorithm or similar
- Writing of paper (about 8 pages, Latex required, template available on website)
- Presentation:
  - 30 min topical presentation
  - discussion

Deadlines

- Deadline for understanding the topic: 6 weeks before presentation
- Deadline for paper draft: 3 weeks before presentation
- Final deadline for everything: date of presentation
- Dates for presentations will be announced during the kick-off

Geometric Multigrid

$\rightarrow$ See paper "Why multigrid methods are so efficient"(I. Yavneh, 2006)

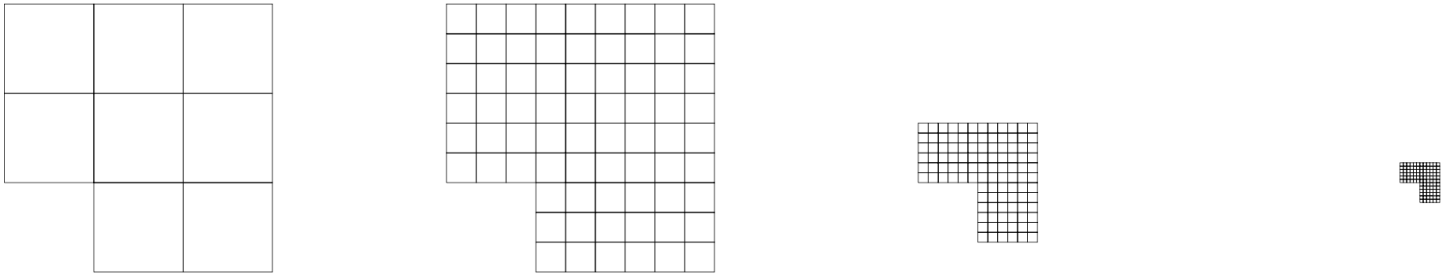
Multigrid for non-linear problems

- Correction scheme: correct only error on coarse grid

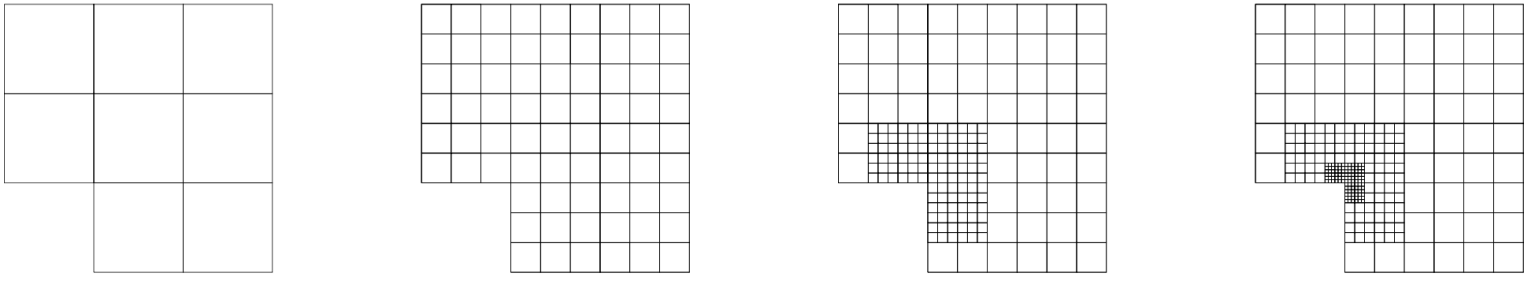
$$\mathrm{L}_k\left(\tilde{u}_k + \hat{u}_k\right) = f_k \Rightarrow \mathrm{L}_k \hat{u}_k = \mathrm{L}_k \tilde{u}_k - f_k = r_k$$

- Does not work with non-linear operators anymore
- Restrict solution onto coarse grid
- Extrapolation with special right hand side possible

Adaptive multigrid



(a) Hierarchy of uniform grids on different domains.



(b) The corresponding composite grids.

Hierarchical Basis Multigrid

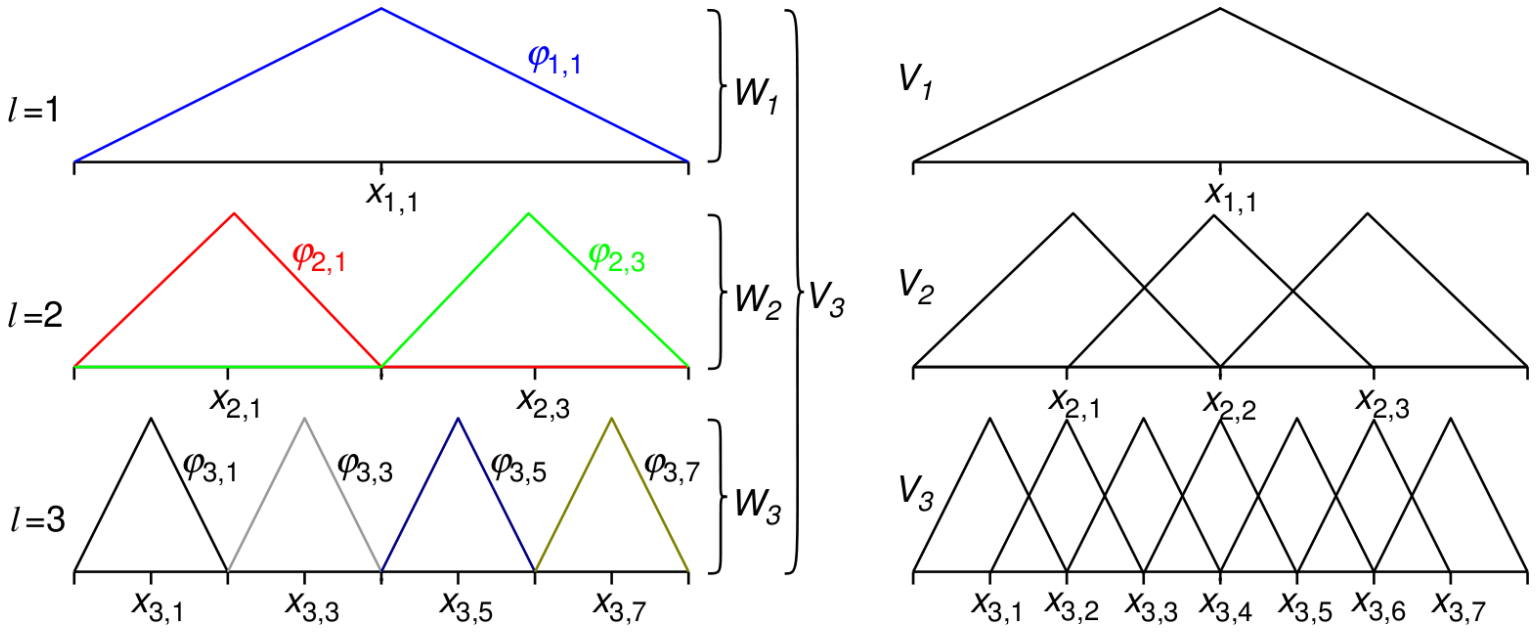


Figure: D. Pflüger: Spatially Adaptive Sparse Grids for High-Dimensional Problems. Verlag Dr. Hut, München, August 2010.

Semidefinite Systems

- Multigrid can be seen as a standard iterative method over semidefinite systems

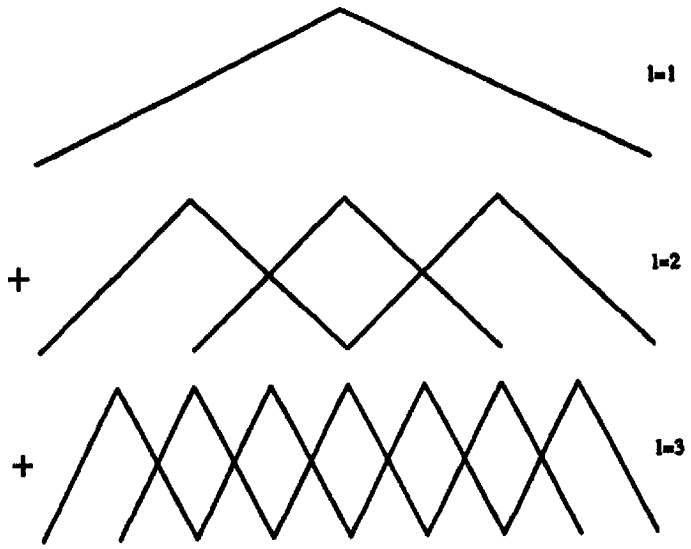


Figure: Griebel, M., Multilevel Algorithms Considered as Iterative Methods on Semidefinite Systems, SIAM Journal on Scientific Computing 1994 15:3, 547-565

Basic Ideas of Algebraic Multigrid (AMG)

- No explicit knowledge of problem geometry
- Working only on matrix entries of system matrix
- Algebraic smoothness ($\hat{=}$ slow to converge)
- Operator-dependent interpolation
- Definition of coarse system without rediscretisation (e.g. Galerkin)

$\rightarrow$ For unstructured problems

Classical AMG

- Brandt, McCormick, Ruge 1982; Stüben 1983; Ruge, Stüben 1987
- Strength of connection
- Chosing the coarse grid (graph algorithm)
- Defining interpolation

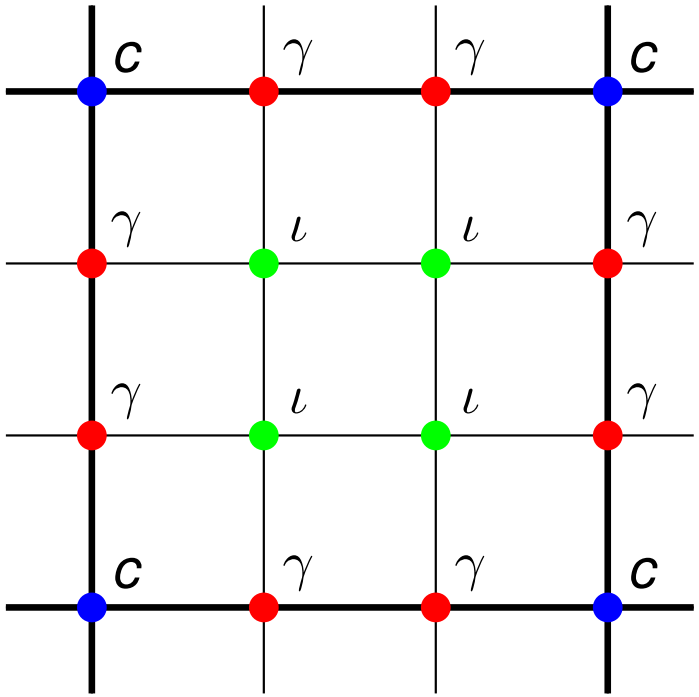
Smoothed Aggregation

- Brezina et al. 2004
- Decompose grid into disjoint subsets of vertices
- Apply tentative prolongation operator (e.g. piecewise constant)
- Smooth output to obtain actual prolongation operator

$$P = (I - \omega D^{-1} A)\tilde{P}$$

BoxMG

- Dendy 1982
- Distinction of point types:



- c points: interpolation weight 1
- γ points: 2 equations in 2 unknowns (by approximation)
- ι points: 4 equations in 4 unknowns

Further Topics

- Smoothers for Multigrid Methods
- Smoothing Analysis
- Parallel Smoothers
- Parallel Geometric Multigrid
- Parallel AMG

References and Further Reading

- W.L. Briggs, V.E. Henson, S.F. McCormick: A Multigrid Tutorial, Second Edition (2000)
- U. Trottenberg, C. Oosterlee, A. Schüller: Multigrid (2001)
- K. Stüben: Algebraic Multigrid (AMG): An Introduction with Applications (1999)
- V.E. Henson: An Algebraic Multigrid Tutorial (1999)
- R.D. Falgout: An Introduction to Algebraic Multigrid, CiSE (2006)