Algorithms for Uncertainty Quantification

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Summer Semester 2017
Lecture 1: Introduction to Uncertainty Quantification
Uncertainty Quantification definition
What is Uncertainty Quantification?

Short definition

UQ = (probability theory + statistical practice) + “real world”
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Short definition

UQ = (probability theory + statistical practice) + “real world”

Wikipedia definition

“Uncertainty Quantification is the science of quantitative characterization and reduction of uncertainties in applications. It tries to determine how likely certain outcomes are if some aspects of the system are not exactly known"
What is Uncertainty Quantification (2)

Definition of Karen Willcox & Youssef Marzouk (The Princeton Companion to Applied Mathematics):

"UQ involves the quantitative characterization and management of uncertainty in a broad range of applications. It employs both computational models and observational data, together with theoretical analysis. UQ encompasses many different tasks, including uncertainty propagation, sensitivity analysis, statistical inference and model calibration, decision making under uncertainty, experimental design, and model validation. UQ thus draws upon many foundational ideas and techniques in applied mathematics and statistics (e.g., approximation theory, error estimation, stochastic modeling, and Monte Carlo methods) but focuses these techniques on complex models—for instance, of physical or socio-technical systems—that are primarily accessible through computational simulation. UQ has become an essential aspect of the development and use of predictive computational simulation tools."
What is Uncertainty Quantification (3)

- new and hot research topic in computational science
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- new and hot research topic in computational science
- to obtain reliable computational results → uncertainty needs to be incorporated a-priori
- UQ = large-scale computational problems
- UQ = challenging; smart techniques are needed to overcome the computational challenges
- in the lecture, we focus on (but don’t limit ourselves to) forward propagation → how do uncertainties propagate through a model?
Forward vs. inverse problems
Uncertainty quantification: forward propagation

1. A. Stuart: Uncertainty Quantification in Bayesian Inversion
Uncertainty quantification: forward propagation

- direction: input $\rightarrow$ model $\rightarrow$ output
- number of uncertain parameters $\Rightarrow$ stochastic dimensionality

\footnote{A. Stuart: Uncertainty Quantification in Bayesian Inversion}

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Uncertainty quantification: Inverse problems
Uncertainty quantification: Inverse problems

- direction: output (data) → model → input
- input domain size ⇒ stochastic dimensionality

\(^2\) A. Stuart: Uncertainty Quantification in Bayesian Inversion
Illustrative example
UQ - An Example

- Classical transonic flow problem
- $M_\infty = 0.734$
- $\alpha = 2.79^\circ$
- $Re = 6.5 \times 10^6$

Figure: RAE 2822 Airfoil

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Sources: Witteveen et al. *Comparison of Stochastic Collocation methods for UQ of the Transonic RAE 2822 airfoil*
UQ - An Example

- Classical transonic flow problem
- $M_\infty = 0.734$
- $\alpha = 2.79^\circ$
- $Re = 6.5 \times 10^6$

Figure: Wall pressure distribution
UQ - An Example

- Introduce uncertainties: $M_\infty = 0.734 \pm 0.005$, $\alpha = 2.79^\circ \pm 0.1$, $t/c = 0.1211 \pm 0.005$.
- Assume a certain distribution, propagate, and analyze the results.

Figure: Surface pressure distribution using uniformly distributed input uncertainties.
UQ - An Example

- Supposing the parameters are uniformly distributed...
UQ - An Example

• ...or gaussian distributed: different results!
UQ - An Example

- Deterministic solution differs from the mean value calculation:
Course description
Prerequisites

- basic knowledge of probability theory and statistics
- basic knowledge of numerical programming
- basic knowledge of computer programming
Covered material

1. L1: Introduction
2. L2: Repetition of probability theory and statistics
3. L3: Sampling methods for UQ I: classical Monte Carlo sampling
4. L4: Sampling methods for UQ II: quasi Monte Carlo sampling
5. L5: Repetition of numerical interpolation and quadrature
7. L7: Intrusive polynomial chaos: Stochastic Galerkin
8. L8: Sparse grids in UQ
9. L9: Sensitivity analysis
10. L10: Random fields approximation in UQ
11. L11: Software for UQ
12. L12: Guest lecturer: Bayesian inverse problems
ECTS, Modules, Tutorials

ECTS, Modules

- 5 ECTS (2+2 lectures/tutorials per week)
- CSE: elective course
- Computer Science, Computer Science: Games Engineering, Data Engineering and Analytics: elective/Master catalogue
- others?
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Tutorials:

- Wed., 12:15-13:45, MI 02.07.023
- tutor: Ionuț Farcaș
Literature