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Algorithms for Uncertainty Quantification

Tutorial 9: Sobol' indices for global sensitivity analysis

In this worksheet, we focus on the variance-based global sensitivity analysis. In particular, we focus on the computation of the Sobol' indices for global sensitivity analysis.

Variance-based global sensitivity analysis

Variance-based sensitivity analysis provides a way to numerically quantify the contribution (take individually or in combination thereof) of uncertain inputs into output uncertainty; as the name suggests, the measure for output uncertainty is the output variance. The uncertain inputs' contribution to the total resulted variance can be measured via the so called (total) Sobol' indices for global sensitivity analysis.
In `chaospy` it is very easy to compute the first order, second order, or total Sobol' indices for global sensitivity analysis. In the following code snippet, we assume that *model_gpc_approx* denotes the polynomial chaos approximation of the underlying model (obtained via `chaospy`'s infrastructure, i.e. *model_gpc_approx* = *cp.fit_quadrature*(...).

```
import chaospy as cp

...

# create the polynomial chaos approximation of the model
model_gpc_approx = cp.fit_quadrature( ... )

# assume that the underlying input distribution is distr
# first order Sobol' indices for global sensitivity analysis
first_order_Sobol_ind = cp.Sens_m(model_gpc_approx, distr)
```

```
# second order Sobol' indices for global sensitivity analysis
second_order_Sobol_ind = cp.Sens_m2(model_gpc_approx, distr)
# total Sobol' indices for global sensitivity analysis
total_Sobol_ind = cp.Sens_t(model_gpc_approx, distr)
```

Assignment 1

Consider the model problem, the linear damped oscillator

$$\begin{cases} \frac{d^2y}{dt^2}(t) + c\frac{dy}{dt}(t) + ky(t) = f\cos(\omega_O t) \\ y(0) = y_0 \\ \frac{dy}{dt}(0) = y_1. \end{cases} \tag{1}$$

Let $t \in [0, 20]$, $\Delta t = 0.01$. The output of interest is $y(10)$. Assume that $c \sim \mathcal{U}(0.08, 0.12)$, $k \sim \mathcal{U}(0.03, 0.04)$, $f \sim \mathcal{U}(0.08, 0.12)$, $y_0 \sim \mathcal{U}(0.45, 0.55)$, $y_1 \sim \mathcal{U}(-0.05, 0.05)$ and $\omega_O = 1.0$. Write a `python` + `chaospy` program to propagate the uncertainty in (c, k, f, y_0, y_1) through the model in Eq. (1) using the generalized polynomial chaos expansion. Asses the expansion coefficients using the pseudo-spectral approach. Consider both non-sparse and sparse 5D pseudo-spectral computation of the coefficients, constructed on Gaussian nodes. To generate multi-variate quadrature nodes and weights consider $K = 5$ (for the 1D quadrature rule); to construct multi-variate orthogonal polynomials, consider $N = 3$ (for the 1D orthogonal polynomials). Compute the first order and total Sobol' indices for global sensitivity analysis in both cases. What do you observe? Repeat the above experiment assuming that $c = 0.1$, $k \sim \mathcal{U}(0.03, 0.04)$, $f \sim \mathcal{U}(0.08, 0.12)$, $\omega_O \sim \mathcal{U}(0.80, 1.20)$, $y_0 \sim \mathcal{U}(0.45, 0.55)$, $y_1 \sim \mathcal{U}(-0.05, 0.05)$. What do you observe?