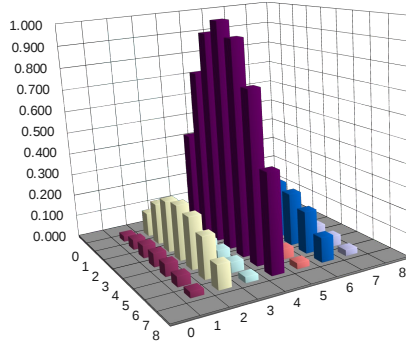




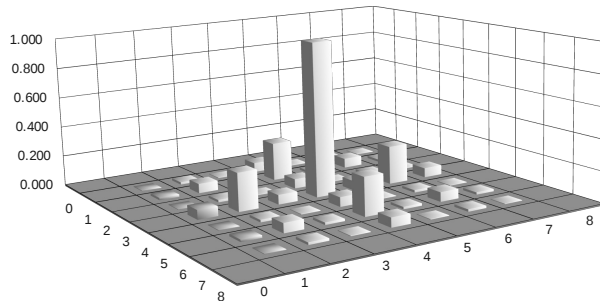
1	0.000	0.027	0.109	0.027	0.438	0.027	0.109	0.027	0.000
2	0.000	0.047	0.188	0.047	0.750	0.047	0.188	0.047	0.000
3	0.000	0.059	0.234	0.059	0.938	0.059	0.234	0.059	0.000
4	0.000	0.063	0.250	0.063	1.000	0.063	0.250	0.063	0.000
5	0.000	0.059	0.234	0.059	0.938	0.059	0.234	0.059	0.000
6	0.000	0.047	0.188	0.047	0.750	0.047	0.188	0.047	0.000
7	0.000	0.027	0.109	0.027	0.438	0.027	0.109	0.027	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000



In row 4 we recognize the known surpluses for  $4 \times x_1 \times (1-x_1)$ , all of the other rows contain multiples of them (the reason lies in the structure of the tensor product of  $f(x_1, x_2) = f_1(x_1) \times f_2(x_2)$ )

2. loop ( $j=1$ ) analogous with rows and columns switched (and, of course, applied to the 1d-surpluses instead of  $f$ )

	0	1	2	3	4	5	6	7	8
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.000	0.004	0.016	0.004	0.063	0.004	0.016	0.004	0.000
2	0.000	0.016	0.063	0.016	0.250	0.016	0.063	0.016	0.000
3	0.000	0.004	0.016	0.004	0.063	0.004	0.016	0.004	0.000
4	0.000	0.063	0.250	0.063	1.000	0.063	0.250	0.063	0.000
5	0.000	0.004	0.016	0.004	0.063	0.004	0.016	0.004	0.000
6	0.000	0.016	0.063	0.016	0.250	0.016	0.063	0.016	0.000
7	0.000	0.004	0.016	0.004	0.063	0.004	0.016	0.004	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000



These are the hierarchical surpluses  $v_{(l_1, l_2), (i_1, i_2)}$  – only with different numbering

### Pagoda bases

In the  $h_1 \times h_2$  we compute the associated weights (in this case 1/4 of the pagodas' bases)

	0	1	2	3	4	5	6	7	8
0									
1		0.016	0.031	0.016	0.063	0.016	0.031	0.016	0.125
2		0.031	0.063	0.031	0.125	0.031	0.063	0.031	0.250
3		0.016	0.031	0.016	0.063	0.016	0.031	0.016	0.125
4		0.063	0.125	0.063	0.250	0.063	0.125	0.063	0.500
5		0.016	0.031	0.016	0.063	0.016	0.031	0.016	0.125
6		0.031	0.063	0.031	0.125	0.031	0.063	0.031	0.250
7		0.016	0.031	0.016	0.063	0.016	0.031	0.016	0.125
8									
		0.125	0.250	0.125	0.500	0.125	0.250	0.125	

### Pagoda volumes

Nun bekommen wir das Pagodenvolumen als Höhe (Überschuss  $v_{(1,1,2),(1,1,2)}$ ) mal  $h_1 \cdot h_2$

Note: This only applies for inner grid points as boundary surpluses are all zero :-)

	0	1	2	3	4	5	6	7	8
0									
1		0.00006	0.00049	0.00006	0.00391	0.00006	0.00049	0.00006	
2		0.00049	0.00391	0.00049	0.03125	0.00049	0.00391	0.00049	
3		0.00006	0.00049	0.00006	0.00391	0.00006	0.00049	0.00006	
4		0.00391	0.03125	0.00391	0.25000	0.00391	0.03125	0.00391	
5		0.00006	0.00049	0.00006	0.00391	0.00006	0.00049	0.00006	
6		0.00049	0.00391	0.00049	0.03125	0.00049	0.00391	0.00049	
7		0.00006	0.00049	0.00006	0.00391	0.00006	0.00049	0.00006	
8									
Summing up column-wise: ( $F_1(D_2)$ for fix $x_2$ )		0.00513	0.04102	0.00513	0.32813	0.00513	0.04102	0.00513	0.43066

The red box to the right (sum over sums of columns) contains the final result!

Computing the absolute error:

Exact solution: 0.44444  
 Subtracting approximation: -0.43066  
0.01378

I1+I2-1	Anzahl Gitter	Punkte pro Gitter	Punkte alle Gitter	Volumen: eine Pagode	Volumen: alle Pagoden	Volumen: Summe	Fehler	Gitterpunkte insgesamt
1	1	1	1	2.50000E-01	2.50000E-01	2.50000E-01	1.94444E-01	1
2	2	2	4	3.12500E-02	1.25000E-01	3.75000E-01	6.94444E-02	5
3	3	4	12	3.90625E-03	4.68750E-02	4.21875E-01	2.25694E-02	17
4	4	8	32	4.88281E-04	1.56250E-02	4.37500E-01	6.94444E-03	49
5	5	16	80	6.10352E-05	4.88281E-03	4.42383E-01	2.06163E-03	129
6	6	32	192	7.62939E-06	1.46484E-03	4.43848E-01	5.96788E-04	321
7	7	64	448	9.53674E-07	4.27246E-04	4.44275E-01	1.69542E-04	769
8	8	128	1024	1.19209E-07	1.22070E-04	4.44397E-01	4.74718E-05	1793
9	9	256	2304	1.49012E-08	3.43323E-05	4.44431E-01	1.31395E-05	4097
10	10	512	5120	1.86265E-09	9.53674E-06	4.44441E-01	3.60277E-06	9217
11	11	1024	11264	2.32831E-10	2.62260E-06	4.44443E-01	9.80165E-07	20481
12	12	2048	24576	2.91038E-11	7.15256E-07	4.44444E-01	2.64910E-07	45057
13	13	4096	53248	3.63798E-12	1.93715E-07	4.44444E-01	7.11944E-08	98305
14	14	8192	114688	4.54747E-13	5.21541E-08	4.44444E-01	1.90404E-08	212993
15	15	16384	245760	5.68434E-14	1.39698E-08	4.44444E-01	5.07053E-09	458753
16	16	32768	524288	7.10543E-15	3.72529E-09	4.44444E-01	1.34524E-09	983041
17	17	65536	1114112	8.88178E-16	9.89530E-10	4.44444E-01	3.55713E-10	2097153
18	18	131072	2359296	1.11022E-16	2.61934E-10	4.44444E-01	9.37790E-11	4456449
19	19	262144	4980736	1.38778E-17	6.91216E-11	4.44444E-01	2.46574E-11	9437185
20	20	524288	10485760	1.73472E-18	1.81899E-11	4.44444E-01	6.46749E-12	19922945