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Lehrstuhl für Informatik V
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SS 2010
Sheet 1
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Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

1 Project: Interpolation of the Trajectory of the Asteroid Pallas[1]

Motivated by the discovery of the asteroids Ceres (1801) and Pallas (1802), Carl Friedrich Gauss studied the computation of planet trajectories in the beginning of the 19th century. Thereby, he was faced with the following problem of trigonometric interpolation.

Interpolation of the Asteroid's Trajectory

This data of the trajectory have been available to Gauss:

Ascension θ (in degrees)	0	30	60	90	120	150
Declination X (in minutes)	408	89	-66	10	338	807
Ascension θ (in degrees)	180	210	240	270	300	330
Declination X (in minutes)	1238	1511	1583	1462	1183	804

Since the declination X is periodic with regard to θ, the given trajectory data should be interpolated by the following trigonometric function:

$$X(\theta) = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{2\pi k\theta}{360}\right) + b_k \sin\left(\frac{2\pi k\theta}{360}\right)\right) + a_6 \cos\left(\frac{2\pi \cdot 6\theta}{360}\right) \quad (1)$$

For the data x_l and $\theta_l = 30l$ from the tabular it must hold that $X(\theta_l) = X_l$ for all $l = 0, \ldots, 11$. Thus,

$$X_l = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{\pi kl}{6}\right) + b_k \sin\left(\frac{\pi kl}{6}\right)\right) + a_6 \cos\left(\pi l\right). \quad (2)$$

[1]Project idea and data are taken from: W. L. Briggs, Van Emden Henson, *The DFT – An Owner's Manual for the Discrete Fourier Transform*, SIAM, 1995

Excercise 1 (Maple)

Use Maple to compute the coefficients a_k and b_k. Plot the graph of the interpolated trajectory.

Hint: Maple can be found in the faculty's computer lab (Rechnerhalle) in /usr/local/applic/bin

Excercise 2

The functions cos and sin are axially respectivelly point symmetric to the ascension of 180 degrees. What can be found for the coefficients a_k and b_k, if the following conditions hold:

$$\begin{aligned} X_l = X(\theta_l) &= X(360 - \theta_l) = X_{12-l} \qquad \text{respectivelly} \\ X_l = X(\theta_l) &= -X(360 - \theta_l) = -X_{12-l} \end{aligned}$$

Hint: Which values are allowed for X_0 and $X - 6$ in the case $X_l = -X_{12-l}$?

Excercise 3

Formulate the Fast Fourier Transformation for real data for this problem.

Small Lexicon of Astronomy

Declination: The angle between the celestial body and the celestial equator (projection of the earth equator on the celestial sphere).

(Right) Ascension: The angle between the First Point of Aries (The point where the ecliptic intersects the celestial equator) and the intersection point of the meridian of a celestial body and the celestial equator. Is equvalent to the geographical longitude but is measured to the east on the celestial equator. The units are usually hours, minutes and seconds, where 24 hours are equal to 360°.

2 Two-dimensional Cosine-Transformation

Excercise 4

The JPEG-method computes the coefficients $\widetilde{F}_{kl}$ from the image data f_{nm} using the following formula

$$\widetilde{F}_{kl} = \frac{1}{N \cdot M} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} f_{nm} \cos\left(\frac{\pi k \left(n + \frac{1}{2}\right)}{N}\right) \cos\left(\frac{\pi l \left(m + \frac{1}{2}\right)}{M}\right)$$

Assume you have a procedure that can compute all coefficients $\widetilde{G}_k$, $k = 0, \ldots, N-1$ efficiently, according to the formula

$$\widetilde{G}_k = \frac{1}{N} \sum_{n=0}^{N-1} f_n \cos\left(\frac{\pi k \left(n + \frac{1}{2}\right)}{N}\right).$$

How can you compute the coefficients $\widetilde{F}_{kl}$ using this procedure?