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SS 2010
Sheet 2
5.5.2010

Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

1 Project: Interpolation of the Trajectory of the Asteroid Pallas[1] (Continued)

Interpolation of the Asteroid's Trajectory

Reminder: The observation data

Ascension θ (in degrees)	0	30	60	90	120	150
Declination X (in minutes)	408	89	-66	10	338	807
Ascension θ (in degrees)	180	210	240	270	300	330
Declination X (in minutes)	1238	1511	1583	1462	1183	804

should be interpolated by the following trigonometrical function, so that the interpolation gives an approximation of the trajectory.

$$X(\theta) = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{2\pi k\theta}{360}\right) + b_k \sin\left(\frac{2\pi k\theta}{360}\right)\right) + a_6 \cos\left(\frac{2\pi \cdot 6\theta}{360}\right), \qquad (1)$$

For the data taken from the tabular, X_l and $\theta_l = 30l$, it should hold that $X(\theta_l) = X_l$ for all $l = 0, \ldots, 11$, which derives to

$$X_l = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{\pi kl}{6}\right) + b_k \sin\left(\frac{\pi kl}{6}\right)\right) + a_6 \cos\left(\pi l\right). \qquad (2)$$

Excercise 1

Show that the interpolation problem in equation (2) is equivalent to

[1] Project idea and data are taken from: W. L. Briggs, Van Emden Henson, *The DFT – An Owner's Manual for the Discrete Fourier Transform*, SIAM, 1995

$$X_l = \sum_{k=-5}^{6} c_k e^{i2\pi kl/12}, \tag{3}$$

if for $k = 1, \dots, 5$ a_k and b_k are chosen as $a_k = 2\mathrm{Re}\{c_k\}$ and $b_k = -2\mathrm{Im}\{c_k\}$, while $c_0 = a_0$ and $c_6 = a_6$.

Use the fact that all $X_l \in \mathbb{R}$ and, thus, that $c_{-k} = c_k^*$.

Excercise 2 (Maple)

Equation (3) also comes from an interpolation problem, i.e. the complex interpolation function

$$C(x) = \sum_{k=-5}^{6} c_k e^{ikx} \tag{4}$$

at the supporting points $x_n = 2\pi n/N$.

Use Maple to compute and plot the interpolation function $C(x)$. Use the a_k and b_k from the last worksheet and construct the C_k for all $k = -\frac{N}{2} + 1, \dots, \frac{N}{2}$.

Can $C(x)$ be used to describe the asteroid's trajectory?

Additional Excercises

Excercise 3: The Roots of Unity ω_N^k

Let ω_N be $\omega_N := e^{i2\pi/N}$. Show the following properties:

- ω_N^k is the solution of the equation $z^N = 1$ (for all $k \in \mathbb{Z}$)
- The complex conjugates holds $(\omega_N^k)^* = \omega_N^{-k}$.
- The sequence $\{\omega_N^k\}_{k=-\infty}^{\infty}$ is N-periodic. The sequence $\{\omega_N^{nk}\}_{k,n=-\infty}^{\infty}$ is N-periodic in k and in n.

Excercise 4: Periodicity of the DFT

The DFT and the IDFT shall be defined like in the lecture:

$$f_n = \sum_{k=0}^{N-1} F_k \omega_N^{nk} \qquad F_k = \frac{1}{N} \sum_{n=0}^{N-1} f_n \omega_N^{-nk}.$$

Show that the sequences $\{f_n\}_{n=-\infty}^{\infty}$ and $\{F_k\}_{k=-\infty}^{\infty}$ are N-periodic in n and k.

Find a pair of DFT/IDFT with other boundaries for the indices. Imagine you have a library, offering the algorithm for a "normal" transformation. Write an algorithm for the other pair of DFT/IDFT, that accomplishes the forward- and backward-transformation by means of the library function.

Excercise 5: DFT of Mirrored data

Assume a dataset f_n, $n = 0, \dots, N-1$. What is the difference of the Fourier coefficients for this dataset and the "mirrored" dataset $\widetilde{f_n} := f_{N-n}$?