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Sheet 3
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Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Excercise 1: DFT and „Padding“

A dataset f_n, $n = 0, \ldots, N-1$ is extended by "zeros", which gives the dataset $\hat{f}_n$, $n = 0, \ldots, M-1$, with

$$\hat{f}_n := \begin{cases} f_n & \text{falls} \quad n \le N-1 \\ 0 & \text{falls} \quad N \le n \le M-1 \end{cases}$$

What is the difference between the Fourier coefficients of the original dataset f_n and the Fourier coefficients of the extended one $\hat{f}_n$?

Excercise 2: Discrete Cosine Transformation

We start with a dataset $f_{-N+1}, \ldots, f_N$, which fulfills the following symmetry constraint:

$$f_{-n} = f_n \qquad \text{für } n = 1, \ldots, N-1$$

a) Show that the according Fourier coefficients

$$F_k = \frac{1}{2N} \sum_{n=-N+1}^{N} f_n \omega_{2N}^{-kn} \tag{1}$$

are real values only and can be be written as:

$$F_k = \frac{1}{N} \left(\frac{1}{2} f_0 + \sum_{n=1}^{N-1} f_n \cos\left(\frac{\pi n k}{N}\right) + \frac{1}{2} f_N \cos(\pi k) \right). \tag{2}$$

b) Show that the F_k also show a symmetry.

c) Let `FFT(f,N)` be a procedure that computes the coefficients F_k efficiently (according to equation (1)) from a field `f` which consists of $2N$ values f_n. (The result is written back to field `f`)

Write a short procedure `DCT(g,N)` which uses procedure `FFT` to compute the coefficients F_k for $k = 0, \ldots, N$ from equation (2) for the (non-symmetrical) data $f_0, \ldots, f_N$, stored in the parameter field g.

Excercise 3: Fast Discrete Cosine Transformation

Formulate the butterfly scheme for equation (1) from excercise 2. Divide the dataset f_n of length $2N$ into a dataset $g_n := f_{2n}$, containing all values with an even index, and a dataset $h_n := f_{2n-1}$, with all values with odd index. Which symmetries can be found in g_n and h_n? Of which kind (Cosine/Sine Transformation, DFT with real data) are the according DFTs of length N?