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Lehrstuhl für Informatik V
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Algorithms of Scientific Computing
(Algorithmen des Wissenschaftlichen Rechnens)

Hierarchical Surplus

For $n \geq 0$ we have an array of length $2^n + 1$ with values $u_0, \ldots, u_{2^n}$. We consider these values to be function values $u : [0, 1] \to \mathbb{R}$ at equidistant positions $x_i = 2^{-n} \cdot i, 0 \leq i \leq 2^n$, i.e. each pair of neighboring points has a distance of $h := 2^{-n}$ between them.

For an $i = 2^k \cdot j$ with odd index j we define $h_k := 2^k \cdot h$. The two values associated with indices $l := 2^k \cdot (j - 1)$ and $r := 2^k \cdot (j + 1)$ we call left and right neighbor. Referring to the function domain they are defined at a distance of h_k left and right of x_i.

Furthermore we introduce

$$v_i := u_i - \frac{1}{2}(u_l + u_r)$$

as the *hierarchical surplus* at position x_i. For the points on the boundary we set $v_0 := u_0$ and $v_{2^n} := u_{2^n}$.

- Write two functions that implement the mapping from the u_i to the v_i and vice versa. The mapping is supposed to happen “in place”, i.e. no additional array is to be allocated.
- Test your functions for values $u_i := x_i^N$, $N = 1, 2, 3$.
- Show that for the hierarchical surplus

$$\Delta = u\left(\frac{a+b}{2}\right) - \frac{u(a) + u(b)}{2}$$

of a sufficiently often continuously differentiable function $u : [a, b] \to \mathbb{R}$ the following condition holds:

$$\Delta = \int_a^b k(x) \cdot u''(x)dx, \quad \text{with } k(x) = \begin{cases} \frac{1}{2}(a - x) & \text{for } a \leq x < \frac{a+b}{2} \\ \frac{1}{2}(x - b) & \text{for } \frac{a+b}{2} \leq x < b \\ 0 & \text{else.} \end{cases}$$