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SS 2010
Sheet 7
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Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Combination Technique

Dealing with hierarchical bases often turns out to be sophisticated. On this worksheet we will therefore see how the so-called *combination technique* finds a sparse grid interpolant, that approximates a function on a number of full grids, each consisting only of a "relatively small" number of grid points.

Let $u_{\underline{L}}$ ($\underline{L} \in \mathbb{N}^2$) for a $u : [0,1]^2 \to \mathbb{R}$ the interpolant in $V_{\underline{L}}$ (interpolating piecewise bilinearly at the inner grid points, at the boundary u is assumed to be zero again).

(i) Rewrite $u_{\underline{L}}$ as a (weighted) sum of $w_{\underline{l}} \in W_{\underline{l}}$.

(ii) For $n \in \mathbb{N}$ write

$$\sum_{|\underline{l}|_1 = n+1} u_{\underline{l}}$$

as a weighted sum of $w_{\underline{l}}$ and arrange the summands with respect to $\underline{l}$.

(iii) Use the previous results to give a representation of the sparse grid interpolant

$$u_n^D := \sum_{|\underline{l}|_1 \leq n+1} w_{\underline{l}}$$

as a weighted sum of $u_{\underline{l}}$.

(iv) Assume you are talking to a person who knows how to approximate the volume $F_2(u)$ through the trapezoidal rule (in 2d) with respect to $u_{\underline{l}}$. Give instructions on how to write a program that implements a sparse grid approximation of $F_2(u)$. Remember Archimedes quadrature.

(v) Compare this method with Archimedes quadrature — what are the (dis-)advantages?