

Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Exercise 1: DFT and „Padding“

A dataset $f_n, n = 0, \dots, N-1$ is extended by "zeros", which gives the dataset $\hat{f}_n, n = 0, \dots, M-1$, with

$$\hat{f}_n := \begin{cases} f_n & \text{falls } n \leq N-1 \\ 0 & \text{falls } N \leq n \leq M-1 \end{cases}$$

What is the difference between the Fourier coefficients of the original dataset f_n and the Fourier coefficients of the extended one \hat{f}_n ?

Exercise 2: Discrete Cosine Transformation

We start with a dataset f_{-N+1}, \dots, f_N , which fulfills the following symmetry constraint:

$$f_{-n} = f_n \quad \text{für } n = 1, \dots, N-1$$

a) Show that the according Fourier coefficients

$$F_k = \frac{1}{2N} \sum_{n=-N+1}^N f_n \omega_{2N}^{-kn} \quad (1)$$

are real values only and can be written as:

$$F_k = \frac{1}{N} \left(\frac{1}{2} f_0 + \sum_{n=1}^{N-1} f_n \cos\left(\frac{\pi nk}{N}\right) + \frac{1}{2} f_N \cos(\pi k) \right). \quad (2)$$

b) Show that the F_k also show a symmetry.

Exercise 3: Fast Discrete Cosine Transformation

Formulate the butterfly scheme for equation (1) from exercise 2. Divide the dataset f_n of length $2N$ into a dataset $g_n := f_{2n}$, containing all values with an even index, and a dataset $h_n := f_{2n-1}$, with all values with odd index. Which symmetries can be found in g_n and h_n ? Of which kind (Cosine/Sine Transformation, DFT with real data) are the according DFTs of length N ?