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Sheet 1
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Algorithms of Scientific Computing

1 Project: Interpolation of the Trajectory of the Asteroid Pallas[1]

Motivated by the discovery of the asteroids Ceres (1801) and Pallas (1802), Carl Friedrich Gauss studied the computation of planet trajectories in the beginning of the 19th century. There, he was faced with the following problem of trigonometric interpolation.

Interpolation of the Asteroid's Trajectory

The following data of the trajectory have been available to Gauss:

Ascension θ (in degrees)	0	30	60	90	120	150
Declination X (in minutes)	408	89	-66	10	338	807
Ascension θ (in degrees)	180	210	240	270	300	330
Declination X (in minutes)	1238	1511	1583	1462	1183	804

Since the declination X is periodic with regard to θ, the given trajectory data should be interpolated by the following trigonometric function:

$$X(\theta) = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{2\pi k\theta}{360}\right) + b_k \sin\left(\frac{2\pi k\theta}{360}\right)\right) + a_6 \cos\left(\frac{2\pi \cdot 6\theta}{360}\right) \tag{1}$$

The data X_l and $\theta_l = 30l$ have to satisfy $X(\theta_l) = X_l$ for all $l = 0, \ldots, 11$. Thus,

$$X_l = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{\pi kl}{6}\right) + b_k \sin\left(\frac{\pi kl}{6}\right)\right) + a_6 \cos(\pi l). \tag{2}$$

Maple Demo

Use Maple to compute the coefficients a_k and b_k. Plot the graph of the interpolated trajectory.

Hint: Maple can be found in the faculty's computer lab (Rechnerhalle) in /usr/local/applic/bin

[1]Project idea and data are taken from: W. L. Briggs, Van Emden Henson, *The DFT – An Owner's Manual for the Discrete Fourier Transform*, SIAM, 1995

Excercise 1

Show that the interpolation problem in equation (2) is equivalent to

$$X_l = \sum_{k=-5}^{6} c_k e^{i2\pi kl/12}, \tag{3}$$

if for $k = 1, \ldots, 5$ a_k and b_k are chosen as $a_k = 2\mathrm{Re}\{c_k\}$ and $b_k = -2\mathrm{Im}\{c_k\}$, while $c_0 = a_0$ and $c_6 = a_6$. Use the fact that all $X_l \in \mathbb{R}$ and, thus, that $c_{-k} = c_k^*$.

Maple Demo

Equation (3) also comes from an interpolation problem, i.e. the complex interpolation function

$$C(x) = \sum_{k=-5}^{6} c_k e^{ikx} \tag{4}$$

at the supporting points $x_n = 2\pi n/N$.

Use Maple to compute and plot the interpolation function $C(x)$. Use the a_k and b_k from excercise 1 and construct the C_k for all $k = -\frac{N}{2} + 1, \ldots, \frac{N}{2}$. Can $C(x)$ be used to describe the asteroid's trajectory?

Excercise 2

The functions cos and sin are axially respectivelly point symmetric to the ascension of 180 degrees. What can be found for the coefficients a_k and b_k, if the following conditions hold:

$$\begin{aligned} X_l = X(\theta_l) &= X(360 - \theta_l) = X_{12-l} \qquad \text{respectivelly} \\ X_l = X(\theta_l) &= -X(360 - \theta_l) = -X_{12-l} \end{aligned}$$

Hint: Which values are allowed for X_0 and X_6 in the case $X_l = -X_{12-l}$?

Excercise 3: DFT and „Padding“

A dataset f_n, $n = 0, \ldots, N-1$ is extended by "zeros", which gives the dataset $\hat{f}_n$, $n = 0, \ldots, M-1$, with

$$\hat{f}_n := \begin{cases} f_n & \text{if} \quad n \leq N-1 \\ 0 & \text{if} \quad N \leq n \leq M-1 \end{cases}$$

What is the difference between the Fourier coefficients of the original dataset f_n and the Fourier coefficients of the extended one $\hat{f}_n$?

Small Lexicon of Astronomy

Declination: The angle between the celestial body and the celestial equator (projection of the earth equator on the celestial sphere).

(Right) Ascension: The angle between the First Point of Aries (The point where the ecliptic intersects the celestial equator) and the intersection point of the meridian of a celestial body and the celestial equator. Is equvalent to the geographical longitude but is measured to the east on the celestial equator. The units are usually hours, minutes and seconds, where 24 hours are equal to $360°$.