Technische Universität München
Lehrstuhl für Informatik V
Univ.-Prof. Dr. M. Bader
Dipl.-Inf. G. Buse, M.Sc. K. Rahnema

SS 2012
Sheet 2
30.4.2012

Algorithms of Scientific Computing

Exercise 1

In the last worksheet we showed that the a_k and b_k can be computed by

$$c_k = \frac{1}{12} \sum_{l=0}^{11} X_l e^{-i2\pi kl/12}, \tag{1}$$

i.e. by a DFT.

Use the idea of the Fast Fourier Transformation, to reduce this DFT of length 12 to the computation of some DFTs of length 6 or 3, respectively.

Use the fact that all $X_l \in \mathbb{R}$.

Draw a diagram, that shows the needed computation steps or write an appropriate program (for example in Maple).

Extra-Exercise for Interested People:

Try to use Edson's algorithm or the mentioned ideas for the Fast Real Fourier Transformation for this problem.

Exercise 2: DFT of Mirrored data

Assume a dataset f_n, $n = 0, \ldots, N-1$. What is the difference of the Fourier coefficients for this dataset and the "mirrored" dataset $\widetilde{f}_n := f_{N-n}$?

Exercise 3:Two-dimensional Cosine-Transformation

The JPEG-method computes the coefficients $\widetilde{F}_{kl}$ from the image data f_{nm} using the following formula

$$\widetilde{F}_{kl} = \frac{1}{N \cdot M} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} f_{nm} \cos\left(\frac{\pi k \left(n + \frac{1}{2}\right)}{N}\right) \cos\left(\frac{\pi l \left(m + \frac{1}{2}\right)}{M}\right)$$

Assume you have a procedure that can compute all coefficients $\widetilde{G}_k$, $k = 0, \ldots, N-1$ efficiently, according to the formula

$$\widetilde{G}_k = \frac{1}{N} \sum_{n=0}^{N-1} f_n \cos\left(\frac{\pi k \left(n + \frac{1}{2}\right)}{N}\right).$$

How can you compute the coefficients $\widetilde{F}_{kl}$ using this procedure?

Excercise 4: Discrete Cosine Transformation

We start with a dataset $f_{-N+1}, \ldots, f_N$, which fulfills the following symmetry constraint:

$$f_{-n} = f_n \qquad \text{for } n = 1, \ldots, N-1$$

a) Show that the corresponding Fourier coefficients

$$F_k = \frac{1}{2N} \sum_{n=-N+1}^{N} f_n \omega_{2N}^{-kn} \tag{2}$$

are real values only and can be written as:

$$F_k = \frac{1}{N}\left(\frac{1}{2} f_0 + \sum_{n=1}^{N-1} f_n \cos\left(\frac{\pi n k}{N}\right) + \frac{1}{2} f_N \cos(\pi k)\right). \tag{3}$$

b) Show that the F_k is symmetric too.