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SS 2013
Sheet 2
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Algorithms of Scientific Computing

Change of Schedule: due to the student assembly on Wednesday, the tutorial will replace the lecture on Thursday, 10-12, in room MI 01.10.011.

Exercise 1

The functions cos and sin are axially respectively point symmetric to the ascension of 180 degrees. What can be found for the coefficients a_k and b_k from the last worksheet, if the following conditions hold:

$$\begin{aligned} X_l = X(\theta_l) &= X(360-\theta_l) = X_{12-l} \qquad \text{respectively} \\ X_l = X(\theta_l) &= -X(360-\theta_l) = -X_{12-l} \end{aligned}$$

Hint: Which values are allowed for X_0 and X_6 in the case $X_l = -X_{12-l}$?

Exercise 2

In the last worksheet we showed that the a_k and b_k can be computed by

$$c_k = \frac{1}{12} \sum_{l=0}^{11} X_l e^{-i2\pi kl/12}, \tag{1}$$

i.e. by a DFT.

Use the idea of the Fast Fourier Transformation, to reduce this DFT of length 12 to the computation of some DFTs of length 6 or 3, respectively.

Use the fact that all $X_l \in \mathbb{R}$.

Draw a diagram, that shows the needed computation steps or write an appropriate program (for example in Python).

Exercise 3: DFT of Mirrored data

Assume a dataset f_n, $n = 0, \ldots, N-1$. What is the difference of the Fourier coefficients for this dataset and the "mirrored" dataset $\widetilde{f_n} := f_{N-n}$?

Exercise 4: DFT and „Padding"

A dataset f_n, $n = 0, \ldots, N-1$ is extended by "zeros", which gives the dataset $\hat{f}_n$, $n = 0, \ldots, M-1$, with

$$\hat{f}_n := \begin{cases} f_n & \text{if} \quad n \leq N-1 \\ 0 & \text{if} \quad N \leq n \leq M-1 \end{cases}$$

What is the difference between the Fourier coefficients of the original dataset f_n and the Fourier coefficients of the extended one $\hat{f}_n$?