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Sheet 8
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Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Hierarchization in Higher Dimensions, Combination Technique

Tutorial 8: Monday, June 2nd 2014, Room MI 02.07.023

Exercise 1: Hierarchization in Higher Dimensions

In this exercise we will implement the multi-recursive algorithm for hierarchization of a multi-dimensional regular sparse grid. The structure of the code resembles strongly the one-dimensional case, and so we again have a class (`PagodaFunction`) representing our grid points.

(i) Implement the refinement criterion `MinLevelCriterion` that adds all points up to a specified level to a given grid.
Hint: In your grid traversal, try to avoid multiple visits to the same grid points.

(ii) Implement the function `hierarchize` efficiently using a recursive approach.
Hint: The underlying traversal algorithm can be implemented similar to the one in (i).

(iii) Implement a function to compute the volume of the sparse grid interpolant.

Exercise 2: The Combination Technique – A Different View on Sparse Grids

Dealing with hierarchical bases often turns out to be sophisticated. On this worksheet we will therefore see how the so-called *combination technique* finds a sparse grid interpolant, that approximates a function on a number of full grids, each consisting only of a “relatively small” number of grid points.

Let $u_{\underline{l}}$ ($\underline{l} \in \mathbb{N}^2$) for a $u : [0, 1]^2 \to \mathbb{R}$ the interpolant in $V_{\underline{l}}$ (interpolating piecewise bilinearly at the inner grid points, at the boundary u is assumed to be zero again).

(i) $V_{\underline{l}}$ can be decomposed into a set of subspaces $W_{\underline{l}}$. Accordingly, the interpolant $u_{\underline{l}} \in V_{\underline{l}}$ can be written as a sum of $w_{\underline{l}} \in W_{\underline{l}}$.

Spot the grid associated with $u_{(3,2)}$ in the right part of Figure 1. Identify those subspaces in the left part that are needed to reconstruct $u_{(3,2)}$.

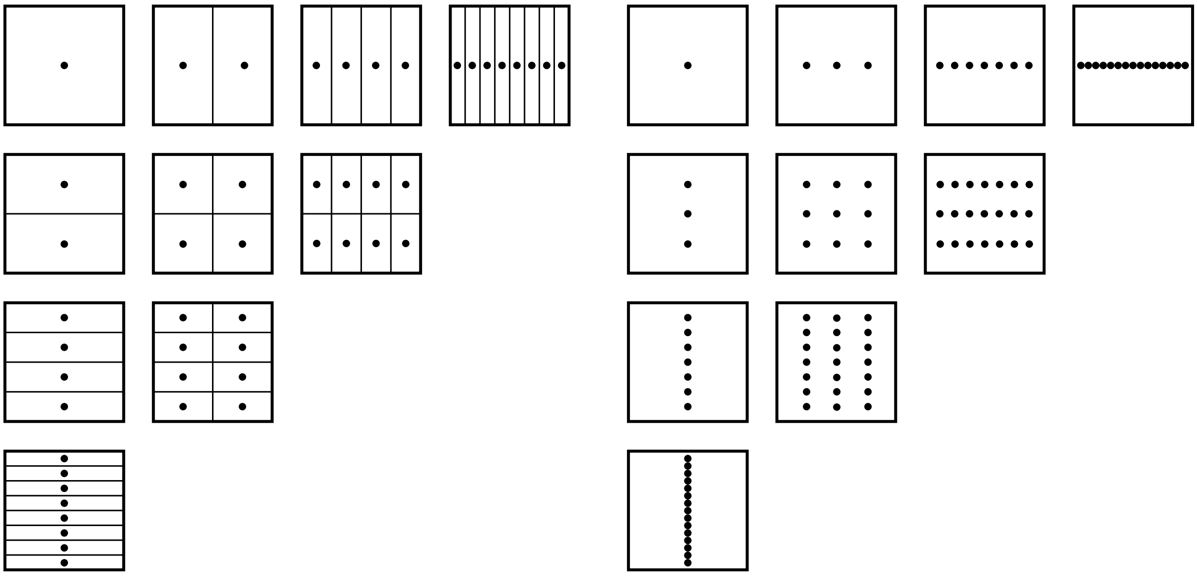

Figure 1: The two parts in the picture show the grid points and supports associated with interpolants $w_{\underline{l}}$ (left) and $u_{\underline{l}}$ (right) up to level 4 for the 2d case.

(ii) Use the result from (i) to rewrite

$$\sum_{|\underline{l}|_1 = n+1} u_{\underline{l}} \,, \qquad n \in \mathbb{N}$$

for the two-dimensional case as a weighted sum of $w_{\underline{l}}$.

Hint: Look at the subspace scheme in Figure 1 and count the occurrences of each subspace in the sum. What do you notice when comparing $w_{\underline{l}}$ with common level $n = |\underline{l}|_1 + dim - 1$?

(iii) In the final step use the previous results to give a representation of the sparse grid interpolant

$$u_n^D := \sum_{|\underline{l}|_1 \leq n+1} w_{\underline{l}}$$

as a weighted sum of $u_{\underline{l}}$. Again, count the occurrences of the $w_{\underline{l}}$.

(iv) Assume you are talking to a person who knows how to approximate the volume $F_2(u)$ through the trapezoidal rule (in 2d) with respect to $u_{\underline{l}}$. Give instructions on how to write a program that implements a sparse grid approximation of $F_2(u)$. Remember Archimedes quadrature.

(v) Compare this method with Archimedes quadrature — what are the (dis-)advantages?