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Sheet 9
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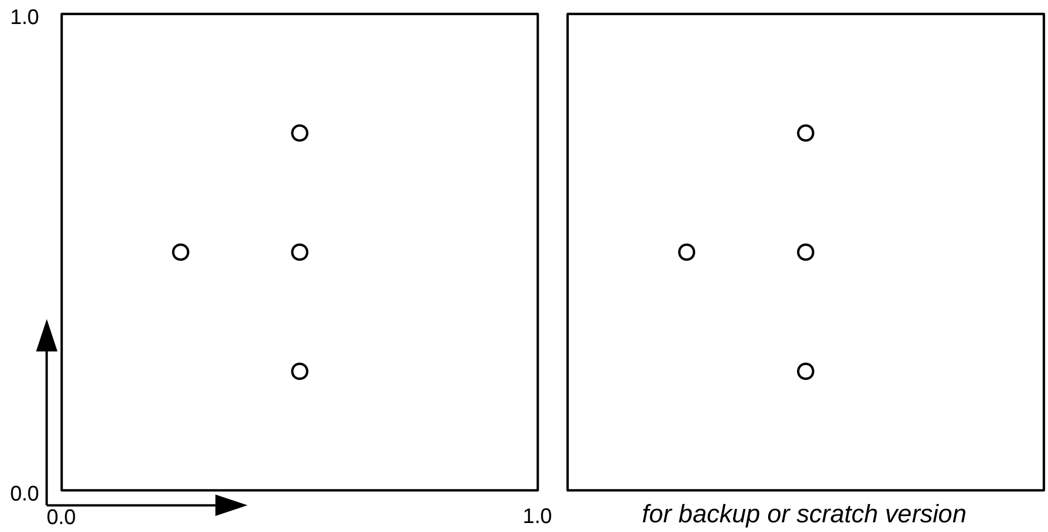
Algorithms of Scientific Computing
(Algorithmen des Wissenschaftlichen Rechnens)
Adaptivity, Norms of Functions

1 Adaptive Sparse Grids

Here, the exercise is to adaptively refine a 2-dimensional sparse grid without boundary. We follow the notation introduced in the lecture and also choose our domain accordingly with $\Omega = [0.0, 1.0]^2$.

1. In the following image you see an incomplete regular sparse grid V_2^1. Insert the missing grid points using small **squares**. What are the level-index-vector pairs $\vec{l}, \vec{i}$ for each of them?



2. Use the (modified) picture from the previous task to perform two steps of adaptive refinement:
   (a) Refine grid point $\vec{l}, \vec{i} = (1, 2), (1, 3)$: create all hierarchical children. Draw its children as small **triangles**. Make sure that you also insert all missing hierarchical parents (and parents of parents, ...) of these children to make the grid suitable for typical algorithms on sparse grids.
   (b) Now refine grid point $(2, 2), (3, 3)$. Again, do not forget to create all missing parents. Draw all new points as small **crosses**.

2 One-dimensional Sparse Grids—An Adaptive Implementation

Last week we introduced Archimedes' approach to approximate the integral $F(f, a, b) = \int_a^b f(x)\ dx$ of a function $f : \mathbb{R} \to \mathbb{R}$, respectively to approximate the function f itself.

For the one-dimensional case we want to formalize this approach and generalize it in the following ways:

- Let $\phi(x)$ be the "mother of all hat functions" with

$$\phi(x) = \begin{cases} x+1 & \text{for } -1 \leq x < 0 \\ 1-x & \text{for } 0 \leq x < 1 \\ 0 & \text{else} \end{cases} \tag{1}$$

- The data structure used to store the hierarchical coefficients is now called *Sparse Grid*.
- A sparse grid is defined by a particular set of interpolation points $x_{l,i}$ and associated ansatz functions $\phi_{l,i}(x)$ with

$$\phi_{l,i}(x) = \phi(\ 2^l \cdot \left(x - i \cdot \frac{1}{2^l}\right)\ ) = \phi(2^l \cdot x - i)\ , \qquad l \in \mathbb{N}^+, i \in \{1, 3, \ldots, 2^l - 1\} \tag{2}$$

- Archimedes' approach from the lecture corresponds to a *regular* sparse grid.
- To improve the quality of approximation for arbitrary functions f we introduce spatial adaptivity.

Your task is to implement the missing parts in the members of the *SparseGrid1d* class and turn it into a fully working adaptive implementation of a one-dimensional sparse grid. Import and use the class *GridPoint* and look at the comments in the provided code snippets for some more details.

a) The constructor *__init__* creates a grid containing all grid points on levels $l \leq minLevel$. A given function f is then evaluated at those points before *hierarchization* is performed eventually to obtain the hierarchical coefficients.
Implement this behavior.

b) Implement the member function *computeVolume* that computes an approximation for $F(f, 0, 1)$ using the current sparse grid interpolant.

c) Implement the member function *refineAdaptively* that takes a certain refinement criterion (see source code) and inserts new grid points accordingly.