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Worksheet 4a
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Algorithms of Scientific Computing

Exercise 1: DFT and Least Square Approximation

Suppose we are given N (odd number) data pairs that we denote (x_n, f_n), where $n = -(N-1)/2 : (N-1)/2$. The x_n are real and are assumed to be equally spaced points on the interval $[-A/2, A/2]$, so $x_n = n\delta x$ where $\delta x = A/N$. The f_n may be complex valued. We want to find an approximation to the data using the N-trigonometric polynomial ϕ_N, given by

$$\phi_N(x) = \sum_{k=-(N-1)/2}^{(N-1)/2} \alpha_k e^{i2\pi kx/A}, \tag{1}$$

the function ϕ_N is called a trigonometric polynomial because it is a polynomial in the quantity $e^{i2\pi x/A}$.

Use the least square criterion, where we minimize the discrete least squares error

$$E = \sum_{n=-(N-1)/2}^{(N-1)/2} |f_n - \phi_N(x_n)|^2, \tag{2}$$

to find N coefficients $\alpha_{-(N-1)/2}, \dots, \alpha_{(N-1)/2}$.

Hint: use the expression for the partial derivative of the error E:

$$\frac{\partial E}{\partial \alpha_k} = \sum_{n=-(N-1)/2}^{(N-1)/2} \left[e^{-i2\pi nk/N} \left(f_n - \sum_{p=-(N-1)/2}^{(N-1)/2} \alpha_p e^{i2\pi np/N} \right) \right], \tag{3}$$

and set these derivatives to 0.

Discrete Sine Transformation
optional exercises (not explained during tutorial)

Exercise 2: Fast Discrete Sine Transformation

Formulate the butterfly scheme for equation

$$F_k = \frac{1}{2N} \sum_{n=-N+1}^{N} f_n \omega_{2N}^{-kn}, \tag{4}$$

where dataset $f_{-N+1}, \dots, f_N \in \mathbb{R}$ fulfills the following symmetry constraint:

$$f_{-n} = -f_n$$

Divide the dataset f_n of length $2N$ into a dataset $g_n := f_{2n}$, containing all values with an even index, and a dataset $h_n := f_{2n-1}$, with all values with odd index. Which symmetries can be found in g_n and h_n? Of which kind (Cosine/Sine Transformation, DFT with real data) are the according DFTs of length N? Which symmetries can be found if the dataset f_n fulfills the following symmetry constraint:

$$f_{-n} = -f_{n+1}.$$