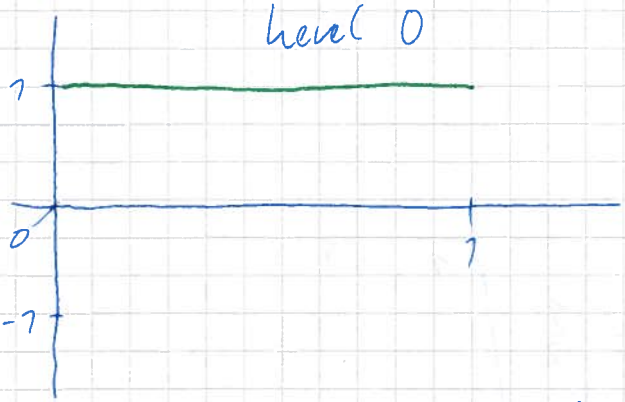
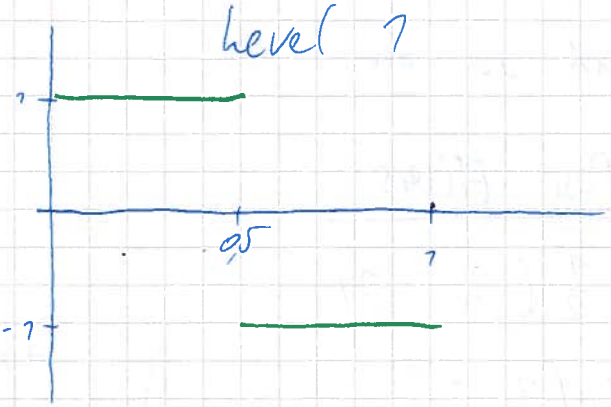
Haar Wavelets - Pyramidal Algorithm.

Graphical representation:

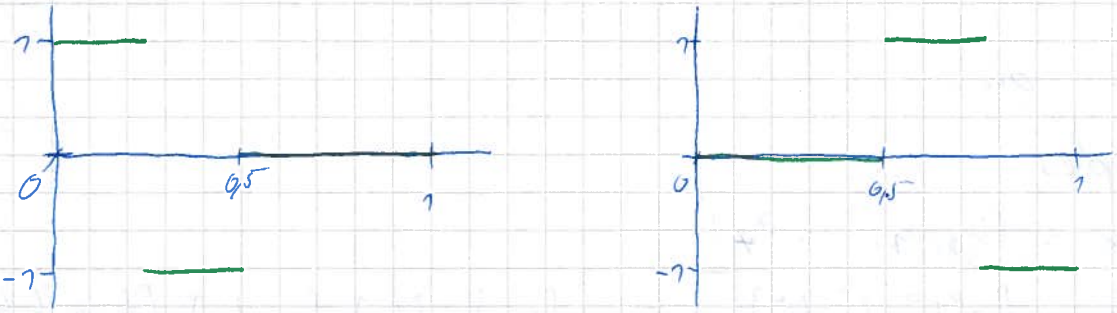
level 0



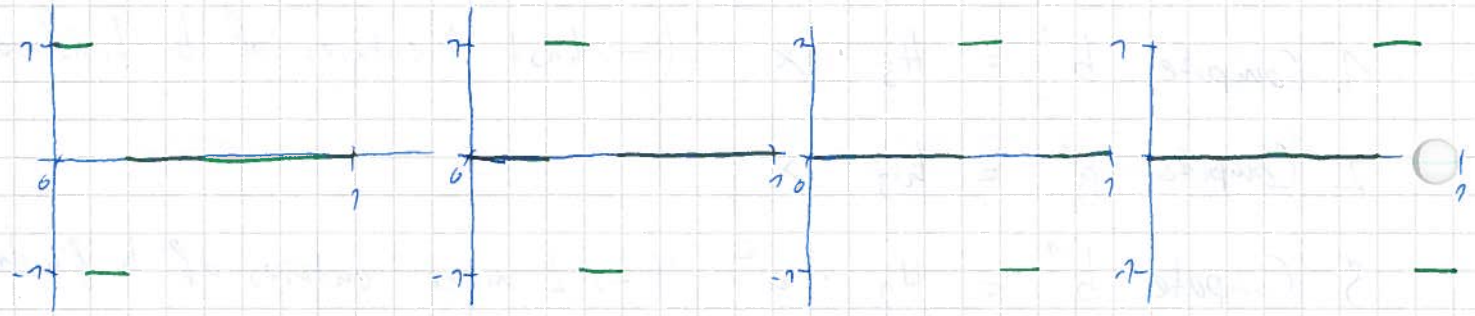
level 1



level 2



level 3



Change of base (until level three):

(we ignore Normalising Coeff. here...)

$$T = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

Transformation Matrix for 8 samples.

Low Pass Filters:

$$h_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$h_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & & \\ & & 1 & 1 \end{pmatrix}$$

$$h_3 = \frac{1}{2}\begin{pmatrix} 1 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & 1 & 1 \end{pmatrix}$$

And so on...

High Pass Filters

$$H_1 = \frac{1}{2}\begin{pmatrix} 1 & -1 \end{pmatrix}$$

$$H_2 = \frac{1}{2}\begin{pmatrix} 1 & -1 & & \\ & & 1 & -1 \end{pmatrix}$$

$$H_3 = \frac{1}{2}\begin{pmatrix} 1 & -1 & & & & \\ & & 1 & -1 & & \\ & & & & 1 & -1 \end{pmatrix}$$

And so on...

For 8 samples:

Input vector: $x = (x_0, x_1, \dots, x_7)^T$

Output vector: $b = [a^0; b^0, b^1, b^2]^T = [[b_0], [b_1], [b_2, b_3], [b_4, b_5, b_6, b_7]]^T$

Sketch of pyramidal algo:

1. Compute $b^2 = H_3 \cdot x$ (→ last 4 entries of b finished)
2. Compute $a^2 = h_3 \cdot x$
3. Compute $b^1 = H_2 \cdot a^2$ (→ 2 more entries of b finished)
4. Compute $a^1 = h_2 \cdot a^2$
5. Compute $b^0 = H_1 \cdot a^1$ (→ 1 more entry of b finished)
6. Compute $a^0 = h_1 \cdot a^1$ (→ last remaining entry of b)

Application for $x = \vec{5} = (1, 2, 3, -1, 1, -4, -2, 4)^T$

1. $b^2 = \begin{pmatrix} -\frac{1}{2} \\ 2 \\ \frac{5}{2} \\ -3 \end{pmatrix}$

2. $a^2 = \begin{pmatrix} \frac{3}{2} \\ 1 \\ -\frac{3}{2} \\ 1 \end{pmatrix}$

3. $b^1 = \begin{pmatrix} \frac{1}{4} \\ -\frac{5}{4} \end{pmatrix}$

4. $a^1 = \begin{pmatrix} \frac{5}{4} \\ -\frac{1}{4} \end{pmatrix}$

5. $b^0 = \begin{pmatrix} \frac{3}{4} \end{pmatrix}$

6. $a^0 = \begin{pmatrix} \frac{1}{2} \end{pmatrix}$

$\rightarrow b = \left(\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, -\frac{5}{4}, -\frac{1}{2}, 2, \frac{5}{2}, -3 \right)^T$