

Algorithms of Scientific Computing

Overview and General Remarks

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Classification of the Lecture

Students of Informatics:

- Informatics Bachelor & Master
- Informatics: Games Engineering (Bachelor)
- Information Systems (Wirtschaftsinformatik)
- Students of Computational Science and Engineering (CSE): application area (cat. E1)

Students of (Computational) Engineering:

- Computational Science and Engineering (CSE): application area (cat. E1)
- Engineering Science

Students of Physics:

various “flavours” → who?

Students of Mathematics:

Bachelor, Master, as minor?

... anyone else? *Warm Welcome!*

Tutorials

Tutors:

- Denis Jarema (FFT, space-filling curves)
- Kilian Röhner (hierarchical methods, sparse grids)

Time & Day:

- by default, tutorials will be on Wednesdays (10-12, MI 00.13.009A)
- starts on April 15 with an introduction to Python/**iPython Notebook**

“Style”:

- worksheets with applications & examples
- no compulsory part

Algorithms in Scientific Computing

Scientific Computing

similar: *Computational Science and Engineering, Wissenschaftliches Rechnen, Simulation-based Science & Engineering, . . .*

Attempt of a definition:

Scientific Computing is . . .

- (numerical) simulation of problems from science or engineering using High Performance Computing (Bungartz, TUM)
- the interdisciplinary conjunction of mathematical and computer science methods as well as different applications of the natural sciences and engineering disciplines, e.g. (TU Darmstadt)
- is concerned with constructing mathematical models and quantitative analysis techniques and using computers to analyze and solve scientific problems (Wikipedia, 2015)
- an *interdisciplinary discipline*
- the focus at our chair SCCS

Algorithms in Scientific Computing?

Central Question: What do I “get” from this lecture?

- ... in particular in the field of Scientific Computing?
- ... in general in the field of Informatics?

⇒ What could/should/do I want to learn in

- ... Informatics?
- ... Computing?

Cross-topical aspects: What central ...

- problems
- techniques, methods
- analytical questions

... of Informatics/Computing/... play a (major) role?

Cross-Topical Aspects

Representation of Information

Claim:

Informatics is the science (or art) of storing information such that it can be used (processed) efficiently.

Examples for information and storage technique:

- tables (data bases of all kind)
- trees, graphs (path searching, . . .)
- objects
- multi-dimensional arrays (raster data, etc.)

Our topic:

How do we store *continuous* data (mathematical functions)?

For Comparison: Representation of Scalars

A brief history of the representation of numbers:

- “tally marks”: |, ||, |||, ||||
(still successfully used to count drinks in bars & restaurants)
- number symbols such as I, V, X, MMIV:
compact but tedious for computing
- positional notation (decimal numbers, binary system, etc.):
ease of arithmetics up to machine computing

Crucial ideas:

- Hierarchy (different “value” of digits depending on their position)
- Structure (concept of 0 as a placeholder!)

Representation of Mathematical Functions

Possibilities of representation (historical):

- *analytical functions*: $f(x) = e^x \sin(x)$
- *tabulated values*
(z.B. logarithm tables, newly rastered data/sampling)
- *interpolation* (also piecewise):
(polygonal chain/curve, polynomial interpolation, spline interpolation, trigonometrical interpolation, ...)

Goal: access and use information efficiently!

- more compact storage
- identification of certain properties (information)
- generally: more efficient algorithms for processing/computations

Our Key Formula: (“coefficients and basis functions”)

$$f(x) \approx \sum c_i \phi_j(x)$$

Multi-Dimensional Data

Examples for multi-dimensional data structures:

- Matrices
- Image data (images, tomography, movies, , ...)
- Discretization based on grids (discretization of physical models / partial differential equations)
- Coordinates of any kind (often going along with graphs)
- Tables (relational databases)
- In financial mathematics: baskets of stocks/options/...

Multi-Dimensional Data

Core topic #1: linearization/sequentialization

- Storage of data structures in memory
- Data processing (traversal)

Demands on linearization (“efficiency”):

- Maintain neighborhood \Rightarrow locality of data, “clustering”
- Simple, fast computation of indices
- “Continuity”, regularity
- Symmetry w.r.t. single dimensions

Space-Filling Curves and Octrees:

- functions on structured adaptive grids
- ordered by space-filling curves

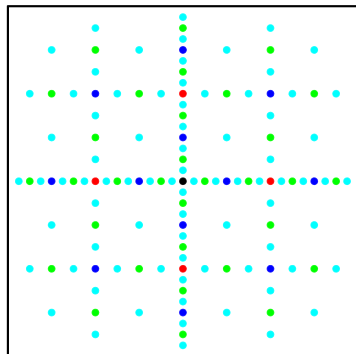
0000	0001	0100	0101
0010	0011	0110	0111
1000	1001	1100	1101
1010	1011	1110	1111

Multi-Dimensional Data

Core topic #2: “curse of dimensionality”

- arrays in 1D, 2D, 3D require n , n^2 , n^3 unknowns
- how about problems, where $n > 3$ (or $n > 10$, or $n > 100$)

“Sparse Grids:”



Recursive Algorithms and Hierarchical Data Structures

“Traditional” style of algorithms in scientific computing:

- FORTRAN programs; procedural/iterative programming
- strongly based on loops and arrays

Nowadays:

- *Recursive and hierarchical*: w.r.t. algorithms (partitioning of problem) and data structures (trees, object orientation)
- *Adaptive*: invest effort, where most benefit can be achieved
- *“Optimal Complexity”*: high approximation order, etc.
- *Distributed*: Computing on parallel and distributed systems
- *Hardware-oriented*: → High Performance Computing

⇒ generally applicable concepts and ideas

Schedule

Fast Fourier Transform:

- discrete Fourier transform as 2D, 3D interpolation
- FFT as divide-and-conquer algorithm
- transform for data compression (images, audio and video data)

Hierarchical basis and sparse grids

- adaptive integration and Archimedes' quadrature
- hierarchical basis functions
- the curse of dimensionality \rightarrow sparse grids
- wavelets

Space trees and space-filling curves

- sequential data structures and traversal of octrees
- definition and construction of space-filling curves
- parallelisation and partitioning