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Worksheet 1
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Algorithms of Scientific Computing

1 Project: Interpolation of the Trajectory of the Asteroid Pallas – Sample Solution

Python Demo

see iPython Notebook solution `Worksheet_1.ipynb`.

Excercise 1

$$X_l = \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}} c_k e^{i2\pi kl/N} = \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}} c_k \cdot \left(\cos\left(\frac{2\pi kl}{N}\right) + i\sin\left(\frac{2\pi kl}{N}\right)\right)$$
$$= \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}} \Re\{c_k\} \cdot \left(\cos\left(\frac{2\pi kl}{N}\right) + i\sin\left(\frac{2\pi kl}{N}\right)\right) + i\Im\{c_k\} \cdot \left(\cos\left(\frac{2\pi kl}{N}\right) + i\sin\left(\frac{2\pi kl}{N}\right)\right)$$

Now we sort for real and imaginery part of X_l:

$$\Re\{X_l\} = \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}} \Re\{c_k\} \cdot \cos\left(\frac{2\pi kl}{N}\right) - \Im\{c_k\} \cdot \sin\left(\frac{2\pi kl}{N}\right)$$
$$\Im\{X_l\} = \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}} \Im\{c_k\} \cdot \cos\left(\frac{2\pi kl}{N}\right) + \Re\{c_k\} \cdot \sin\left(\frac{2\pi kl}{N}\right)$$

Since X_l is real $\Im\{X_l\}$ must be zero. Thus, the remaining part is

$$X_l = \sum_{k=-\frac{N}{2}+1}^{\frac{N}{2}} \Re\{c_k\} \cdot \cos\left(\frac{2\pi kl}{N}\right) - \Im\{c_k\} \cdot \sin\left(\frac{2\pi kl}{N}\right)$$

By reducing the sum to the range $k = 1, \ldots, \frac{N}{2} - 1$, we get

$$X_l = \Re\{c_0\} + \Re\{c_{N/2}\} \cdot \cos(\pi l) + \sum_{k=1}^{\frac{N}{2}-1} \Re\{c_k\} \cdot \cos\left(\frac{2\pi kl}{N}\right) + \Re\{c_{-k}\} \cdot \cos\left(\frac{2\pi(-k)l}{N}\right) - \Im\{c_k\} \cdot \sin\left(\frac{2\pi kl}{N}\right) - \Im\{c_{-k}\} \cdot \sin\left(\frac{2\pi(-k)l}{N}\right)$$

Using the symmetry of the sine and cosine functions, this can be derived to

$$X_l = \Re\{c_0\} + \Re\{c_{N/2}\} \cdot \cos(\pi l) + \sum_{k=1}^{\frac{N}{2}-1} (\Re\{c_k\} + \Re\{c_{-k}\}) \cdot \cos\left(\frac{2\pi kl}{N}\right) + (\Im\{c_{-k}\} - \Im\{c_k\}) \cdot \sin\left(\frac{2\pi kl}{N}\right)$$

Since $c_{-k} = c_k^*$ it is $\Re\{c_{-k}\} = \Re\{c_k\}$ and $\Im\{c_{-k}\} = -\Im\{c_k\}$. So, we get

$$X_l = \Re\{c_0\} + \Re\{c_{N/2}\} \cdot \cos(\pi l) + \sum_{k=1}^{\frac{N}{2}-1} 2\Re\{c_k\} \cdot \cos\left(\frac{2\pi kl}{N}\right) - 2\Im\{c_k\} \cdot \sin\left(\frac{2\pi kl}{N}\right)$$

For $N = 12$, $a_k = 2\Re\{c_k\}, b_k = -2\Im\{c_k\}$ for all $k = 1, \ldots, \frac{N}{2}$, $a_0 = c_0$ and $a_{\frac{N}{2}} = c_{\frac{N}{2}}$ we get equation (2):

$$X_l = a_0 + \sum_{k=1}^{5} \left(a_k \cos\left(\frac{\pi kl}{6}\right) + b_k \sin\left(\frac{\pi kl}{6}\right)\right) + a_6 \cos(\pi l)$$

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