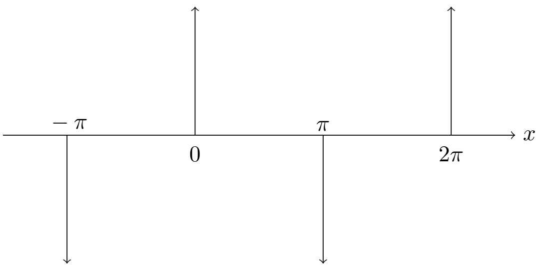
Technische Universität München
Lehrstuhl für Informatik V
Univ.-Prof. Dr. M. Bader
M.Sc. A. Mo-Hellenbrand, M.Sc. A. Schwarz

SS 2016
Worksheet 4
11.5.2016

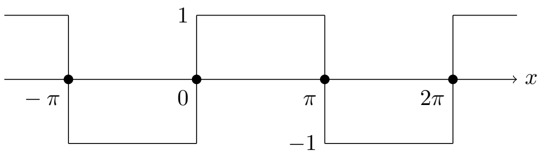
Algorithms of Scientific Computing

Exercise 1: Fourier Series

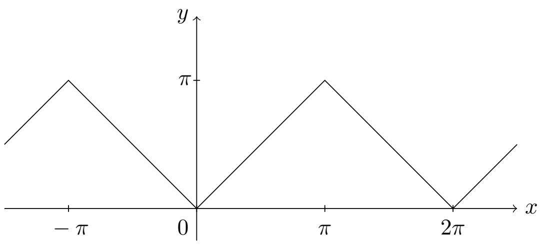
Compute the Fourier coefficients for the following three periodic functions. What is their Fourier series?



a) Repeating Dirac delta.



b) Odd square wave.



c) Repeating ramp.

Exercise 2: DFT and Least Squares Approximation

Let N be an odd number. Suppose we are given N data pairs (x_n, f_n), $n = -(N-1)/2, ..., (N-1)/2$. The x_n are real-valued and evenly spaced in the interval $[-A/2, A/2]$, so $x_n = n\Delta x$ where $\Delta x = A/N$. The f_n may be complex valued. We want to find an approximation to the data using the N-trigonometric polynomial ϕ_N, given by

$$\phi_N(x) = \sum_{k=-(N-1)/2}^{(N-1)/2} \alpha_k e^{i2\pi kx/A}. \tag{1}$$

The function ϕ_N is called a trigonometric polynomial because it is a polynomial in the quantity $e^{i2\pi x/A}$.

Use the least square criterion, where we minimize the discrete least squares error

$$E = \sum_{n=-(N-1)/2}^{(N-1)/2} |f_n - \phi_N(x_n)|^2, \tag{2}$$

to find the N coefficients $\alpha_{-(N-1)/2}, \ldots, \alpha_{(N-1)/2}$. The error is minimized when its gradient vector is zero. Use that the k-th partial derivative is given by

$$\frac{\partial E}{\partial \alpha_k} = \sum_{n=-(N-1)/2}^{(N-1)/2} \left[e^{-i2\pi nk/N} \left(f_n - \sum_{p=-(N-1)/2}^{(N-1)/2} \alpha_p e^{i2\pi np/N} \right) \right]. \tag{3}$$