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SS 2016
Worksheet 5
18.5.2016

Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Numerical Quadrature for One-dimensional Functions

This week we focus on the question how to determine the definite integral

$$F(f,a,b) = \int_a^b f(x)\,dx \quad \text{for functions } f : [a,b] \to \mathbb{R}.$$

It is not always possible to integrate the function f analytically, and so we use numerical quadrature. For the implementation of our algorithms we will use the Python language. You will find an IPython notebook in your profile on the server containing more detailed information on the single exercises.

Excercise 1: Analytical Integration

Consider the functions

$$f(x) = -4\,x\,(x-1) \tag{1}$$

$$g(x) = \frac{8}{9} \cdot (-16\,x^4 + 40\,x^3 - 35\,x^2 + 11\,x). \tag{2}$$

Compute the antiderivatives and evaluate the integrals.

Hint: If not specified otherwise, the domain considered from now on is the unit interval $\Omega = [0,1]$.

Excercise 2: Composite Trapezoidal Rule

Write a function that approximates the integral via the Composite Trapezoidal Rule.

Excercise 3: Composite Simpson Rule

Do the same as in Exercise 2, only use the Composite Simpson Rule now.

Excercise 4: Archimedes' Hierarchical Approach

Again we focus on the question how to determine the definite integral

$$F(f,a,b) = \int_a^b f(x)\,dx \quad \text{for functions } f : [a,b] \to \mathbb{R}.$$

In this exercise we will use Archimedes' approach to approximate the integral.

Let $\vec{u} = [u_0, \ldots, u_n]^T \in \mathbb{R}^n, n = 2^l - 1, l \in \mathbb{N}$ a vector of function values with $u_i = f(x_i = \frac{i+1}{2^l})$.

a) Write a function that transforms a given vector $\vec{u} \in \mathbb{R}^n$ to a similar vector $\vec{v} \in \mathbb{R}^n$ containing the hierarchical coefficients needed for Archimedes' quadrature approach. **Hint:** Later in the lecture we will officially call this process "hierarchization", thus the function name.

b) Having computed the vector $\vec{v}$ with the hierarchical coefficients, implement a function evaluating the integral.

c) Write a function "dehierarchize1d" similar to "hierarchize1d" that computes the inverse of the transformation above.

Excercise 5: Thoughts about Adaptivity

Discuss how the previous methods could be extended in order to improve their approximation quality.