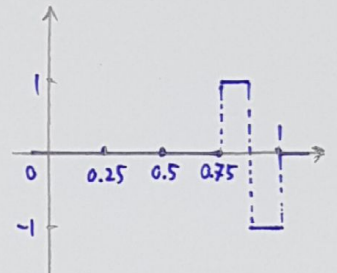
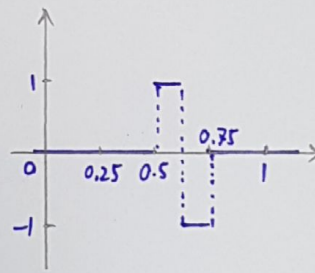
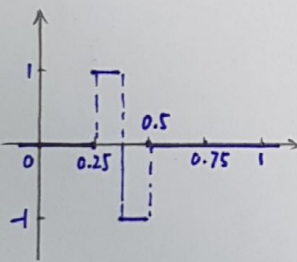
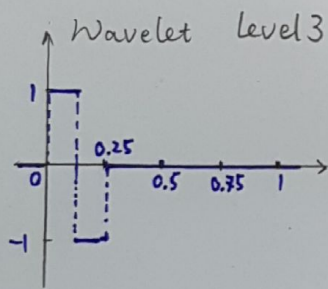
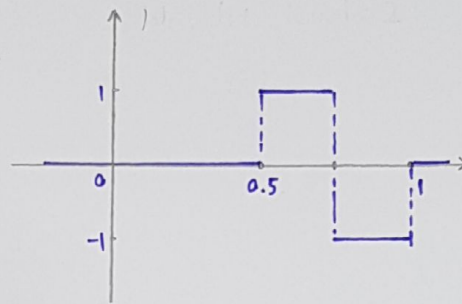
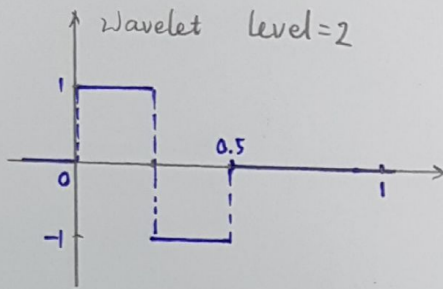
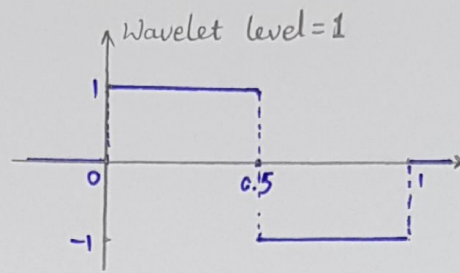
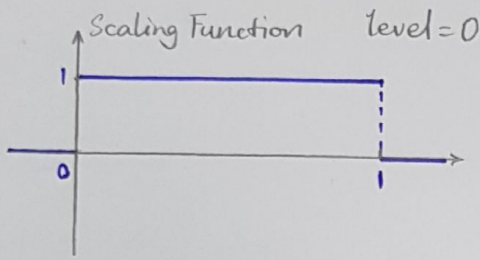


# Haar Wavelet Basis Transform: Pyramidal Algorithm

[Graphical Representation]



Transformation matrix (level 3):

$$\mathcal{M} = \begin{matrix}
 \begin{matrix}
 \psi = \\
 \varphi_0
 \end{matrix}
 & \begin{matrix}
 1 \\
 1 \\
 1 \\
 1 \\
 1 \\
 1 \\
 1 \\
 1
 \end{matrix}
 & \begin{matrix}
 1 \\
 1 \\
 -1 \\
 -1 \\
 -1 \\
 -1 \\
 0 \\
 -1
 \end{matrix}
 & \begin{matrix}
 1 \\
 1 \\
 -1 \\
 -1 \\
 0 \\
 0 \\
 0 \\
 0
 \end{matrix}
 & \begin{matrix}
 0 \\
 0 \\
 0 \\
 0 \\
 1 \\
 1 \\
 0 \\
 0
 \end{matrix}
 & \begin{matrix}
 1 \\
 -1 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{matrix}
 & \begin{matrix}
 0 \\
 0 \\
 1 \\
 -1 \\
 0 \\
 0 \\
 0 \\
 0
 \end{matrix}
 & \begin{matrix}
 0 \\
 0 \\
 0 \\
 0 \\
 1 \\
 -1 \\
 0 \\
 0
 \end{matrix}
 & \begin{matrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 1 \\
 -1
 \end{matrix}
 \end{matrix}
 \end{matrix}$$

Input Signal Vector :  $S = [1, 2, 3, -1, 1, -4, -2, 4]^T$

Low-pass Filter:

$$h_1 = \frac{1}{2} [1 \ 1]$$

$$h_2 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$h_3 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

High-pass Filter:

$$H_1 = \frac{1}{2} [1 \ -1]$$

$$H_2 = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$H_3 = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$a^3 = S$$

$$b^2 = H_3 a^3 = \left[-\frac{1}{2}, 2, \frac{5}{2}, -3\right]^T$$

$$a^2 = h_3 a^3 = \left[\frac{3}{2}, 1, -\frac{3}{2}, 1\right]^T$$

$$b^1 = H_2 a^2 = \left[\frac{1}{4}, -\frac{5}{4}\right]^T$$

$$a^1 = h_2 a^2 = \left[\frac{5}{4}, -\frac{1}{4}\right]^T$$

$$b^0 = H_1 a^1 = \frac{3}{4}$$

$$a^0 = h_1 a^1 = \frac{1}{2}$$

Output Vector :  $b = [a^0, b^0, b^1, b^2]$

$$= \left[\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, -\frac{5}{4}, -\frac{1}{2}, 2, \frac{5}{2}, -3\right]^T$$

$\underbrace{\hspace{1.5cm}}_{a^0} \quad \underbrace{\hspace{1.5cm}}_{b^0} \quad \underbrace{\hspace{1.5cm}}_{b^1} \quad \underbrace{\hspace{1.5cm}}_{b^2}$