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Worksheet 7
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Algorithms of Scientific Computing (Algorithmen des Wissenschaftlichen Rechnens)

Wavelets and Finite Element Method

Exercise 1: Function approximation

In problems of function interpolation or regression, we are given a set of data points $\{(y_1,x_1),\ldots,(y_m,x_m)\}$, we seek for a function $f \in \mathbb{V}$, such that the distance between this function and all data points are minimized, e.g.,

$$f(x) = \arg\min_f \left\{ \sum_{j=1}^{m} \|y_j - f(x_j)\| \right\}. \tag{1}$$

We can express f in the form of

$$f(x) = \sum_i \alpha_i \cdot \phi_i(x), \tag{2}$$

where $\{\phi_i\}$ is a set of known basis functions that span the function space, i.e., $\mathbb{V} = \text{span}\{\phi_i\}$. Now the problem of searching for f has become searching for $\{\alpha_i\}$, the coefficients of basis functions.

In this exercise, we would like to employ the same strategy to approximate a function $g(x)$. The goal is to look for an f of the form (2) that can best approximate $g(x)$, e.g.,

$$f(x) = \arg\min_f \|g(x) - f(x)\|_2. \tag{3}$$

1. Show that solving (3) is equivalent to finding a set of coefficients $\{\alpha_k\}$, where

$$\int_{-\infty}^{+\infty} \phi_k(x) \left(g(x) - \sum_i \alpha_i \cdot \phi_i(x) \right) \mathbf{d}x \stackrel{!}{=} 0,\ \forall k \tag{4}$$

2. Transform (4) into a linear system of the form $A\alpha = b$. What does A look like?
3. Let $g(x) = -4x^2 + 4x,$ where $x \in [0,1]$. Solve (4) with nodal basis hat functions and piecewise constant functions.

Exercise 2: Discrete Wavelet Transform

Compute the DWT for the Haar wavelets for the signal $s = [8,4,-1,1,0,4,1,7,-\frac{5}{2},-\frac{3}{2},0,-4,-2,-2,1,-5]$ using the Pyramidal Algorithm. Discuss the computation complexity of this method.

Exercise 3: Discrete Wavelet Transform 2D

Compute the DWT for the Haar wavelets for the 2D signal $s = \begin{bmatrix} 4 & 2 & 3 & 5 \\ 1 & -7 & 0 & 8 \\ -1 & -3 & 9 & -3 \\ 6 & -2 & -1 & 1 \end{bmatrix}$.