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SS 2016
Worksheet 8
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Algorithms of Scientific Computing
(Algorithmen des Wissenschaftlichen Rechnens)
Multi-dimensional Quadrature

In order to find an alternative way to approximate the d-dimensional integral

$$F_d := \int_{[0,1]^d} f(x_1, \ldots, x_d)\, d(x_1, \ldots, x_d)$$

we use a regular d-dimensional grid of step size $h = 2^{-l}$ stored in an array with indices in $[0, 2^l]^d$.

We don't want to be bothered with special boundary treatment so we simplify this task assuming our function is 0 on the boundary, i.e.

$$\forall \mathbf{x} = [x_1, ..., x_d]^T : \exists j \text{ such that } (x_j = 0 \vee x_j = 1) \;\Rightarrow\; f(\mathbf{x}) = 0$$

In the next step we compute the d-dimensional hierarchical surpluses:

> **loop** $j = 1, \ldots, d$ **over the dimensions**
> **loop over all 1d subgrids discretizing spatial direction** j
> (fix all coordinates except x_j, what you'll get is a 1d array
> of grid points like on worksheet 4 — for $d = 2$ this comes down
> to processing all rows and all columns once)
> **Compute 1d surpluses in dimension** j **(in place) for each subarray**

Once we have the surpluses $v_{\underline{l},\underline{i}}$ (indexing with level $\underline{l}$ and point index $\underline{i}$ as introduced in the lecture) we only need to multiply them with the base area of the associated pagoda and sum everything up. The base area of the pagoda associated with grid point $x_{\underline{l},\underline{i}}$ is $2^{-|\underline{l}|_1}$.

Let

$$f(x_1, x_2) = 16x_1(1-x_1)x_2(1-x_2) \;=\; \left(1 - 4(x_1 - \frac{1}{2})^2\right) \cdot \left(1 - 4(x_2 - \frac{1}{2})^2\right).$$

1. Implement this method!
   For $d = 2$ and $l = 3$ this can be done easily in a spreadsheet (using copy and paste is not only ok but also encouraged for thorough understanding).
2. Write the summed up volumes in descending order (compute with your program or derive from results of worksheet 5). Imagine n to be large.
   How many volumes are at least needed to approximate the integral $(4/9)$ with an absolute error not larger than $1/144$ (this choice is not random, it will give you nice results).
3. Spot and draw the used grid points in the unit square!