Technische Universität München
Lehrstuhl für Informatik V
Univ.-Prof. Dr. M. Bader
M.Sc. J.-M. Gallard, M.Sc. M. Obersteiner

SS 2018
Worksheet 2
18.04.2018

Algorithms of Scientific Computing

Exercise 1

The functions cos and sin are axially respectively point symmetric to the ascension of 180 degrees. What can be found for the coefficients a_k and b_k from the last worksheet, if the following conditions hold:

$$\begin{aligned} X_l = X(\theta_l) &= X(360-\theta_l) = X_{12-l} \qquad \text{respectively} \\ X_l = X(\theta_l) &= -X(360-\theta_l) = -X_{12-l} \end{aligned}$$

Hint: Which values are allowed for X_0 and X_6 in the case $X_l = -X_{12-l}$?

Exercise 2

In the last worksheet we showed that the a_k and b_k can be computed by

$$c_k = \frac{1}{12} \sum_{l=0}^{11} X_l e^{-i2\pi kl/12}, \tag{1}$$

i.e. by a DFT.

Use the idea of the Fast Fourier Transformation, to reduce this DFT of length 12 to the computation of some DFTs of length 6 or 3, respectively.

Use the fact that all $X_l \in \mathbb{R}$.

Draw a diagram, that shows the needed computation steps or write an appropriate program (for example in Python).

Exercise 3: DFT of Mirrored data

Assume a periodic dataset f_n, $n = 0, \ldots, N-1$. What is the difference of the Fourier coefficients of this dataset and the "mirrored" dataset $\widetilde{f}_n := f_{N-n}$ for $n = 1, \ldots, N-1$ and $\widetilde{f}_0 := f_0 \equiv f_N$?

Exercise 4: DFT and "Padding"

A dataset f_n, $n = 0, \dots, N-1$ is extended by "zeros", which gives the dataset $\hat{f}_n$, $n = 0, \dots, M-1$, with

$$\hat{f}_n := \begin{cases} f_n & \text{if} \quad n \leq N-1 \\ 0 & \text{if} \quad N \leq n \leq M-1 \end{cases}$$

What is the difference between the Fourier coefficients of the original dataset f_n and the Fourier coefficients of the extended one $\hat{f}_n$?