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Worksheet 4
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Algorithms of Scientific Computing

Exercise 1: Fast Discrete Cosine Transform

$$F_k = \frac{1}{2N} \sum_{n=-N+1}^{N} f_n \omega_{2N}^{-kn} \tag{1}$$

Determine the butterfly scheme for equation (1) from the previous exercise (worksheet 3). Divide the dataset f_n of length $2N$ (with $f_{-n} = f_n$) into a dataset $g_n := f_{2n}$, containing all values with an even index, and a dataset $h_n := f_{2n-1}$, with all values with an odd index. Which symmetries can be found in g_n and h_n? Of which kind (Cosine/Sine Transformation, DFT with real data) are the according DFTs of length N? Which symmetries can be found if the dataset f_n fulfils the following symmetry constraint:

$$f_{-n} = f_{n+1}.$$

Exercise 2: DFT and Least Square Approximation

For a given $N \in \mathbb{N}$ and $A \in \mathbb{R}$, let $\Delta x = A/N$.

$\forall n, 0 \leq n < N$, $x_n = n\Delta x$ and $f_n \in \mathbb{C}$ form the data pair (x_n, f_n). Note that the x_n are the equally spaced point of the interval $[0, A - \Delta x]$.

We want to find an approximation to the data using the N-trigonometric polynomial ϕ_N, given by

$$\phi_N(x) = \sum_{k=0}^{N-1} \alpha_k e^{i2\pi kx/A}, \tag{2}$$

the function ϕ_N is called a trigonometric polynomial because it is a polynomial in the quantity $e^{i2\pi x/A}$.

The approximation should fit the least square criterion, where we minimize the discrete least squares error E defined as

$$E = \sum_{n=0}^{N-1} |f_n - \phi_N(x_n)|^2 \tag{3}$$

Find the N coefficients $\alpha_0, \ldots, \alpha_{N-1}$. Do you know an algorithm to compute them efficiently ?

Hint: use the expression for the partial derivative of the error E:

$$\frac{\partial E}{\partial \alpha_k} = \sum_{n=0}^{N-1} \left[e^{-i2\pi nk/N} \left(f_n - \sum_{p=0}^{N-1} \alpha_p e^{i2\pi np/N} \right) \right], \tag{4}$$

and set these derivatives to 0.

Exercise 3: Fast Discrete Sine Transform

Formulate the butterfly scheme for the equation

$$F_k = \frac{1}{2N} \sum_{n=-N+1}^{N} f_n \omega_{2N}^{-kn}, \tag{5}$$

where the dataset $f_{-N+1}, \ldots, f_N \in \mathbb{R}$ fulfils the following symmetry constraint:

$$f_{-n} = -f_n$$

Split the dataset f_n of length $2N$ into a dataset $g_n := f_{2n}$, containing all values with an even index, and a dataset $h_n := f_{2n-1}$, with all values with an odd index. What symmetries can be found in g_n and h_n?