Technische Universität München
Institut für Informatik
Prof. Dr. Hans-Joachim Bungartz
Daniel Butnaru, M. Sc.
Dipl.-Inf. Wolfgang Eckhardt

WS 2010/2011

Algorithms of Scientific Computing II

Exercise 6 - Norms and Interpolation with Sparse Grids with respect to the Energy Norm

In the lecture, sparse grids were obtained by considering the maximum and the L2 norms. In this exercise we mostly repeat the most important steps and additionally consider the energy norm. To not go into that much detail, intermediate results (estimations) from the lecture shall be used. We will look at the bilinear interpolation in one and two dimensions. Similar to the previous exercise, the functions we consider are $f := 4x(x-1)$ and respectively $f := 16x_1(x_1-1)x_2(x_2-1)$. The (pseudo) code from the previous exercise can be used as a basis.

First, we recapitulate the three norms and determine the approximation quality for the above functions f using the full grid $V_n^{(\infty)}$. This is done by constructing the interpolants u_n for $n = 1, \ldots, 10$ and calculating the error $||f - u_n||$ in the three norms (please plot it). What are the convergence orders with respect to h_n?

For the full grid we the have used the subspaces $W_{\boldsymbol{l}}$ with $|\boldsymbol{l}|_\infty \leq n$:

$$V_n^{(\infty)} := \bigoplus_{|\boldsymbol{l}|_\infty \leq n} W_{\boldsymbol{l}}$$

Now we want to pick the better subspaces i.e. those with a cost-benefit ratio smaller or equal to a certain criteria. To identify these better subspaces, we need the following estimations with respect to all norms,

- the cost for a single subspace $c(\boldsymbol{l})$ and
- the benefit of a single subspace $s(\boldsymbol{l})$.

Now we can consider the overall cost-benefit ratio and we want to do this for 1D and 2D:

- For which $W_{\boldsymbol{l}}$ is the ratio constant? Draw (for continuous $\boldsymbol{l}$) the equipotential lines of the cost-benefit ratio.

What are the effects of using different norms on

- the overall costs,
- the overall benefit
- the approximation quality? Calculate and plot the approximation quality as above (for the full grid interpolant) for $n = 1, \ldots, 10$.