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Algorithms of Scientific Computing II

Exercise 7 - Smolyak-Quadrature and Operator Dependent Prolongation

1 Smolyak Quadrature

In this assignment we will perform the Smolyak quadrature on the two dimensional domain $[0,1]^2$. We will use the trapezoidal rule as the nested univariate quadrature rule $Q_n^{(1)}$.

- Write a method, which returns for a given p and l the 1D-trapezoidal rule (weights) for p^l intervals.
- Write a function which implements the difference formula $\Delta_l^{(1)}$. The function takes as parameters p, l and a function to build the 1D quadrature rule (the above function) and returns a quadrature rule (weights).
- Next, implement the tensor product $\otimes$, which takes two 1D quadrature rules and builds a 2D rule.
- Now we can build the Smolyak quadrature by programming the sum on slide 97.

2 Operator Dependent Prolongation

Let us solve the ODE:

$$-u''(x) + k^2 \cdot u(x) = f(x) \tag{1}$$

(k $\in \mathbb{R}$) on the unit interval $[0,1]$, divided in N ($N \geq 4$) equal subintervals.

- What are the solutions of the homogeneous ($f \equiv 0$) (undiscretized) equation?
- How does the finite difference stencil for the differential operator look like?
- How does the linear equation system look like for the boundary problem with Dirichlet boundary conditions $u(0) = u_0, u(1) = u_1$?
- Construct a prolongation operator which derives from the values of a coarse grid (even index) the values on the finer grid (odd index) so that the interpolant in the finer grid points fulfill the homogeneous differential equation.
- what do we still need to finish our multigrid solver for the LES?