Technische Universität München
Institut für Informatik
Scientific Computing in Computer Science

WS 2013/14
Worksheet
09.12.2013

Fundamental Algorithms 8

Exercise 1

Write an algorithm that copies all keys that are stored in a binary search tree into an array of appropriate size. In the resulting array, the keys shall be sorted in descending order.

Solution:

The main idea of the algorithm is to rely on recursion and:

1. copy all keys of the right subtree to the array (in sorted order, using the algorithm recursively);
2. copy the root of the tree into the array;
3. copy all keys of the left subtree to the array (in sorted order, using the algorithm recursively).

Using pseudo code, the algorithm can be written as:

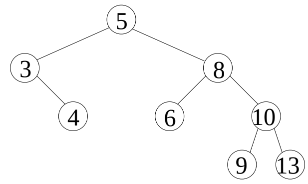
```
Tree2Array(T: BinTree, A: Array[1..n], pos:Integer) : Integer {
   /* write elements of the binary search tree T into array A,
      starting at position pos;
      return index of next empty element in A
    */
    if (T != emptyTree) { /* if T is not an empty tree */
       pos := Tree2Array(T.rightSon,A,pos);
       A[pos] := T.key; pos := pos + 1;
       pos := Tree2Array(T.leftSon,A,pos);
    };
    return pos;
}
```

Exercise 2

Consider the binary tree given by the expression

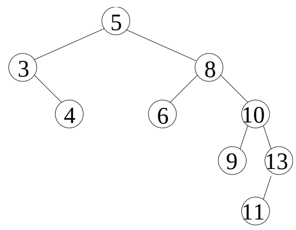
```
x = (5, (3, emptyTree, (4, emptyTree, emptyTree)),
     (8, (6, emptyTree, emptyTree), (10, (9, emptyTree, emptyTree),
     (13, emptyTree, emptyTree))))
```

- draw a diagram of this binary tree and decide whether its a binary search tree

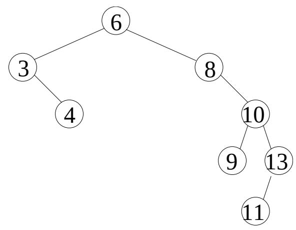


  For each node, all keys in the left subtree are smaller than that in the node, and all keys in the right subtree are larger. Hence, the tree is a binary search tree.

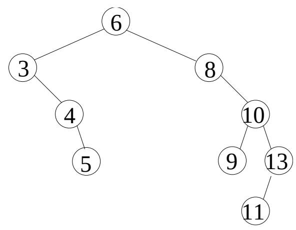
- perform the following operations (using the resp. algorithms from the lectures), and draw a diagram of the search tree after each operation:
  - `TREE_INSERT(x,11)`



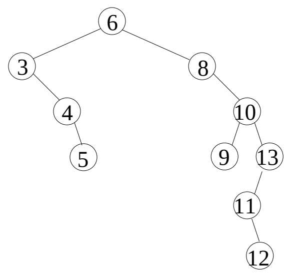
  - `TREE_DELETE(x,5)`



  - `TREE_INSERT(x,5)`



  - `TREE_INSERT(x,12)`



Exercise 3

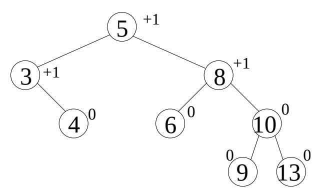
Decide whether the binary tree given in exercise IV is an AVL tree

- before the insert/delete operations, and
- after each of the regular insert/delete operations.

Again, perform the insert/delete operations given in exercise IV, and name and perform the rotation(s) to restore the AVL property after each step (if required). Draw a diagram of the search tree after each of your insert/delete, or rotation operations.

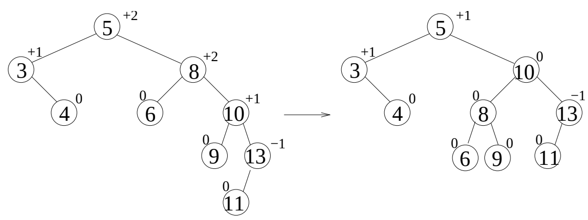
Solution:

Before the insert/delete operations, the height balances for the nodes are:

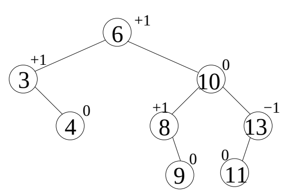


Therefore, the binary search tree is also an AVL tree.

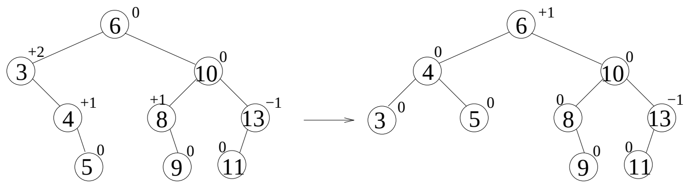
- after `TREE_INSERT(x,11)`, the AVL property is violated in node 8 $\rightarrow$ left-rotation on node 8:



- after `TREE_DELETE(x,5)`, we still have an AVL tree $\rightarrow$ rotation required:



- after `TREE_INSERT(x,5)`, the AVL property is violated in node 3, which requires a left-rotation on node 3:



- after `TREE_INSERT(x,12)`, the AVL property is violated in node 13, and a left-right-rotation is required: